OBERSEMINAR ALGEBRA UND GEOMETRIE
WS 2014/15
TOPIC: ESSENTIAL DIMENSION

Roughly speaking, the *essential dimension* of some "algebraic object" is the minimal number of transcendental parameters truly needed to define the object. For example, the set of similarity classes of $n \times n$-matrices seems to depend on n^2 parameters, but the rational canonical form and companion matrices reveal that exactly n parameters are needed.

Introduced by Buhler and Reichstein [1997] for finite groups, essential dimension was generalized to algebraic groups [Reichstein 2000], by Merkurjev to functors, as exposed in [Berhuy and Favi 2008] and [Merkurjev 2013], and finally to algebraic stacks [Brosnan, Reichstein and Vistoli 2011]. For elementary overviews, see [Beauville 2012] and [Reichstein 2012]. The seemingly elementary notion of essential dimension has many surprising relations to various fields of algebra and algebraic geometry. Even for finite groups, the computation of its essential dimension over the complex numbers is usually an open question.

The goal of this Oberseminar topic is to learn the fundamental notions and results in the theory of essential dimension. The plan is to go slowly through the overview [Berhuy and Favi 2008] section-wise. In the second half, we have some talks on more specialized results discussed in [Merkurjev 2013].

Talk 1: Galois cohomology [Berhuy and Favi 2008], Section 2

Talk 2: Cohomological invariants [Berhuy and Favi 2008], Section 3

Talk 3: Free actions and torsors [Berhuy and Favi 2008], Section 4

Talk 4: Versal pairs and Rost's definition [Berhuy and Favi 2008], Section 5

Talk 5: Generic torsors and compressions [Berhuy and Favi 2008], Section 6

Talk 6: Some finite groups [Berhuy and Favi 2008], Section 7

Talk 7: Finite groups of essential dimension one [Ledet 2007]

Talk 8: Finite groups of small essential dimension [Beauville 2011]

Talk 8: Essential dimension of complex abelian varieties [Brosnan 2007]

Talk 9: Upper bound for essential dimension of PGL_{p^s} [Ruozzi 2011]

Talk 10: Essential dimension of PGL_{p^2} I [Merkurjev 2010]

Talk 11: Essential dimension of PGL_{p^2} II [Merkurjev 2010]

Beauville 2011: Finite simple groups of small essential dimension. Preprint, arXiv: 1101.1372, 7 pages, 2011.

Beauville 2012: De combien de paramètres dpend l'equation générale de degré n? Gaz. Math. No. 132, 5–15.

Berhuy and Favi 2003: Essential dimension: a functorial point of view (after A. Merkurjev). Doc. Math. 8, 279–330.

Brosnan 2007: The essential dimension of a g-dimensional complex abelian variety is 2g. Transform. Groups 12 (2007), 437–441.

Brosnan, Reichstein and Vistoli 2011: Essential dimension of moduli of curves and other algebraic stacks. J. Eur. Math. Soc. 13, 1079–1112.

Buhler and Reichstein 1997: On the essential dimension of a finite group. Compositio Math. 106, 159–179.

Ledet 2007: Finite groups of essential dimension one. J. Algebra 311, 31–37.

Merkurjev 2009: Essential dimension. In: R. Baeza, W. Chan, D. Hoffmann, R. Schulze-Pillot (eds.), Quadratic forms–algebra, arithmetic, and geometry, pp. 299–325. Amer. Math. Soc., Providence, RI.

Merkurjev 2010: Essential p-dimension of $\mathrm{PGL}(p^2)$. J. Amer. Math. Soc. 23, 693–712.

Merkurjev 2013: Essential dimension: a survey. Transform. Groups 18, 415–481.

Reichstein 2000: On the notion of essential dimension for algebraic groups. Transform. Groups 5, 265–304.

Reichstein 2010: Essential dimension. In: R. Bhatia, A. Pal, G. Rangarajan, V. Srinivas, M. Vanninathan (eds.): Proceedings of the International Congress of Mathematicians. Volume II, pp. 162–188. Hindustan Book Agency, New Delhi.

Reichstein 2012: What is ... essential dimension? Notices Amer. Math. Soc. 59, 1432–1434.

Ruozzi 2011: Essential p-dimension of PGL_n. J. Algebra 328 (2011), 488–494.