

ADVANCED SEMINAR ON GROUP THEORY – SOSE 2019
TWISTED CONJUGACY CLASSES, REIDEMEISTER ZETA
FUNCTIONS AND DYNAMICAL SYSTEMS

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The objective of the seminar is to study twisted conjugacy classes, Reidemeister numbers and Reidemeister zeta functions from a group-theoretic point of view. For an overview of the subject see the report [2], which is however pitched in a slightly different direction and also no longer up-to-date in some regards. Selected more recent developments are surveyed in [3], but also this article is now almost ten years old.

Some classical theorems, e.g., Pontryagin duality, will be covered. But mainly we will focus on understanding selected aspects of the work of Fel'shtyn and others over the past 20 years. The talks are more or less independent of each other, but – taken together – should form a bigger picture. Inevitably, some details will need to be skipped here or there.

We currently plan to have 10 talks, but this can be adjusted during the course of the seminar. Depending on the participants' interests, it would be possible to add further individual talks on selected topics.

Here is a short outline of the planned talks.

Talk 1: Introduction, overview and basic definitions.

Cover the definitions of twisted conjugacy classes, Reidemeister numbers, Reidemeister zeta function, give examples (in particular, finite groups). Sources: [2, 3, 5].

Talk 2: Pontryagin duality.

Discuss and prove (sketch) the classical Pontryagin duality. Sources: [2, Section 2.2], [5, Sec. 1.2] for an overview; [16, Ch. 1] for details; [14, Sec. 2.9] for Pontryagin duality for profinite groups.

Talk 3: Rationality results for Reidemeister zeta functions.

Cover rationality theorems of Reidemeister zeta functions, possibly also functional equations, in special situations, e.g. eventually commutative endomorphisms, finitely generated Abelian groups. Sources: [5, 6] and [2, Sec. 2.6].

Talk 4: Twisted conjugacy classes in nilpotent groups.

Cover twisted conjugacy problem in nilpotent groups, and give definitions of the Reidemeister spectra of free nilpotent groups, support for Reidemeister zeta functions. Source: [15].

Talk 5: Twisted Burnside theorems for polycyclic-by-finite-groups I.

Coordinate with Talk 6. The goal is to prove the twisted analogue of the Burnside theorem for polycyclic-by-finite-groups. Sources: [9, 10, 11].

Talk 6: Twisted Burnside theorems for polycyclic-by-finite-groups II.

Coordinate with Talk 5. Sources: [9, 10, 11].

Talk 7: Reidemeister numbers for Baumslag–Solitar groups.

Give a short introduction into Baumslag–Solitar groups and follow the sources for results on their Reidemeister numbers. Sources: [4], and [12] for a generalisation.

Talk 8: Twisted conjugacy classes in weakly branch groups.

Give a short introduction into weakly branch groups and follow the sources for results on their Reidemeister numbers in special cases. Sources: [7, 17].

Talk 9: The R_∞ -property for Chevalley groups.

Make suitable selection of results. Sources: [8, 13]

Talk 10: Dynamical zeta functions associated to compact connected abelian groups.

Cover: explicit formula, Pólya–Carlson dichotomy, connection with Reidemeister zeta function. Source: [1].

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