Problems for Tutorial 1

(Thursday, 18.11, at 10 a.m.)

For computation of integrals, you can use Google. Until Monday, you can solve Problems 1-2, after Monday you can solve Problems 3-4.

Problem 1. Prove Lemma 1.2.6 from the script:

(a) For any real r, there exists a transformation from $\text{Möb}_{\mathbb{R}}$, which sends $\mathbf{A}_r$ to $\mathbf{A}_0$. [2 P.]

(b) For any two different reals r_1, r_2, there exists a transformation from $\text{Möb}_{\mathbb{R}}$, which sends $\mathbf{C}_{r_1,r_2}$ to $\mathbf{A}_0$. [5 P.]

Problem 2.

(a) Compute the hyperbolic length of the Euclidean segment from $1+i$ to $3+3i$. [3 P.]

(b) Compute the hyperbolic distance $\rho(1+i, 3+3i)$ between the points $1+i$ and $3+3i$. [3 P.]

Compare the numbers in (a) und (b) with the help of a calculator.

Problem 3.

(a) Let C be the Euclidean circle with the center $(3+2i)$ and the radius 1. Compute the hyperbolic length of C. [6 P.]

(b) Let A be the hyperbolic circle in $\mathbb{H}$ with the center $a+ib$ and the hyperbolic radius r. Let B be the Euclidean circle in $\mathbb{H}$ with the center $a+ib\,\mathrm{ch}(r)$ and of Euclidean radius $b\sqrt{\mathrm{ch}^2(r)-1}$. Prove that $A=B$. [4 P.]

(c) Compute the hyperbolic radius and the hyperbolic center of the hyperbolic circle C. [3 P.]

Hint. Formula (2) from Theorem 1.3.8 in the script can be written in the following form:

$$\mathrm{ch}\,\rho(a+ib, x+iy) = 1 + \frac{(x-a)^2+(y-b)^2}{2by}.$$

Prove the above statement (b) with the help of this formula.

Problem 4. Let ABC be the hyperbolic triangle, where $A=i$, $B=\sqrt{5}i$, $C=2+i$.

(a) Compute the hyperbolic angles of this triangle. For that you can use the function `arctan`. [5 P.]

(b) Compute the hyperbolic area of this triangle. [1 P.]