

Problems for Tutorial 4

(Thursday, 9.12, at 10 a.m.)

Problem 1. Prove that every Fuchsian group is countable.

[5 P.]

Problem 2. Consider

$$A = \begin{pmatrix} 5 & -3 \\ \frac{9}{2} & -\frac{5}{2} \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}.$$

For $X \in \{A, B\}$ define the type of the Möbius transformation T_X , compute $\widehat{\text{Fix}}(T_X)$ and find all T_X -invariant geodesic lines in \mathbb{H} .

[5 P.]

Problem 3. Let $r > 0$. Consider the Möbius transformations

$$\begin{aligned} \theta_r : z &\rightarrow rz, \\ \psi : z &\mapsto -\frac{1}{z}. \end{aligned}$$

Prove that the subgroup $H_r := \langle \theta_r, \psi \rangle$ is discrete in $\text{Möb}_{\mathbb{R}}$.

[5 P.]

Problem 4.

- (a) Consider the metric space (\mathbb{Q}, d) of rational numbers with the usual metric. Prove that \mathbb{Q} does not have the Heine-Borel property, i.e. there is a closed and bounded subset S of \mathbb{Q} which is not compact.
- (b) Let (\mathbb{H}, ρ) be the hyperbolic metric space. Prove that a subset S of \mathbb{H} is compact if and only if it is bounded and closed.

[3 P.]

[5 P.]