

# ULTRAPRODUCTS IN COMMUTATIVE ALGEBRA

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The goal of this reading group is to study ultralimits and related topics in commutative algebra.

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**TALK 1: Bounds in the theory of polynomial rings over fields.** *Chapter Section 4 of [1].*

This talk introduces some “standard” proofs based on ultraproducts. These are due to Schmidt and Van den Dries in their seminal paper “Bounds in the theory of polynomial rings over fields. A nonstandard approach”.

*Plan:* Review basic concepts from the first chapters. Prove theorems 4.4.1, 4.4.2, 4.4.3 and 4.4.4 of [1].

**TALK 2: Ultraproducts in strong Artin approximation.** *Chapter 7 of [1].*

We say that a local ring  $(S, \mathcal{M})$  has the **strong Artin approximation property** if any system of polynomial equations with coefficients in  $S$  which is solvable in  $S$  modulo  $\mathcal{M}^m$  for all  $m$  is already solvable in  $S$ . One of the main aims of this talk is Theorem 7.1.9 of [1]: the strong Artin approximation property holds in the ring of power series over an algebraically closed *ultra-field* with several variables.

*Plan:* Prove Theorems 7.1.8, 7.1.9 and 7.1.10 of [1].

**TALK 3: Cataproducts.** *Chapter 8 of [1].*

Noetherianity is a very desirable property for a ring. Unfortunately, an ultraproduct of Noetherian rings is not necessarily Noetherian. In this talk we introduce a variation of the ultraproduct, the *cataproduct*, and we show that a cataproduct of Noetherian local rings (under some extra assumption) is complete and Noetherian (Theorem 8.1.4).

*Plan:* Define cataproducts. Prove Theorem 8.1.4 and ancillary lemmas.

## References

[1] Hans Schoutens. The Use of Ultraproducts in Commutative Algebra.

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