



Figure 1: Action of $SL_2(\mathbb{Z})$ on upper half plane

By !Original:Kilom691Vector: Alexander Hulpke - Own work based on: ModularGroup-FundamentalDomain-01.png, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=59963451>

generators of $SL_2\mathbb{Z}$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$z \mapsto -\frac{1}{z}$$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

translation

$$z \mapsto z+1$$

fundamental domain



$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ acts trivial on \mathbb{H}

$$S^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(ST)^3 = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

S has fixed pt i

ST has fixed pt

$$-\frac{1}{2} + i\frac{\sqrt{3}}{2} = w$$

Can retract fund domain to the path on unit circle

moving around the arc

connecting i and w get a tree \mathcal{T}

w/ $SL_2\mathbb{Z}$ -action