Group theory days in Düsseldorf - Talks

Equations in acylindrically hyperbolic groups - Algebraic, verbal and existential closedness

Oleg Bogopolski – Heinrich-Heine-Universität Düsseldorf

I will explain basic notions, give some examples, and present new results in these areas.

Between Tits buildings and free factor complexes

Benjamin Brück – Universität Bielefeld

Much of the modern treatment of automorphism groups of free groups is motivated by analogies with arithmetic groups. I will present a new family of complexes interpolating between two well-studied objects associated to these classes of groups: the free factor complex and the Tits building of $GL_n(\mathbb{Q})$. Each of these new complexes is associated to the automorphism group $\mathrm{Aut}(A_\Gamma)$ of a right-angled Artin group and has the homotopy type of a wedge of spheres. The dimension of these spheres forms a new invariant associated to $\mathrm{Aut}(A_\Gamma)$.

Conciseness and strong conciseness in profinite groups

Eloisa Detomi – University of Padova

A group word w is said to be concise in a class $\mathcal{C}$ of groups if, for each G in $\mathcal{C}$ such that G_w is finite, also $w(G)$ is finite. For topological groups, especially profinite groups, a variation of the classical notion arises quite naturally: we say that w is strongly concise in a class $\mathcal{C}$ of topological groups if, for each G in $\mathcal{C}$, already the bound $|G_w| < 2^{\aleph_0}$ implies that $w(G)$ is finite.

In this talk we discuss conciseness of some words in the class of all residually finite groups and strong conciseness in the class of all profinite groups. We will show that multilinear commutator words are strongly concise in the class of all profinite groups and that every group word is strongly concise in the class of nilpotent profinite groups. From this we can deduce, for instance, that, if w is one of the group words x^2, x^3, x^6, $[x^3, y]$ or $[x, y, y]$, then w is strongly concise in the class of all profinite groups.

Fibring of manifolds and groups

Dawid Kielak – Universität Bielefeld

I will discuss how one can use a little group homology to reprove and generalise statements about 3-manifolds fibring over the circle.

Isomorphism of nilpotent groups via derivations

Joshua Maglione – Universität Bielefeld

By bringing in tools from multilinear algebra, we introduce a general method to aid in the computation of isomorphism for groups. Of particular interest are nilpotent groups where the only classically known proper nontrivial characteristic subgroup is the derived subgroup. This family of groups poses the biggest challenge to all modern approaches. Through structural analysis of the bi-additive commutator map, we leverage the representation theory of Lie algebras to prove efficiency for families of nilpotent groups. We report on joint work with Peter A. Brooksbank, Uriya First, and James B. Wilson.

On Finitely Presented Groups that Contain $\mathbb{Q}$

Francesco Matucci – University of Milano-Bicocca

It is a consequence of Higman's embedding theorem that the additive group $\mathbb{Q}$ of rational numbers can be embedded into a finitely presented group. Though Higman's proof is constructive, the resulting group presentation would be very large and unpleasant. In 1999, Pierre de la Harpe asked for an explicit and "natural" example of a finitely presented group that contains an embedded copy of $\mathbb{Q}$. In this talk, we describe some solutions to de la Harpe's problem related to Thompson's groups F, T, and V. Moreover, we prove that there exists a group containing $\mathbb{Q}$ which is simple and has type F infinity. This is joint work with Jim Belk and James Hyde.

On Generalisations of the Basilica group

Moritz Petschick & Karthika Rajeev – Heinrich-Heine-Universität Düsseldorf

We will discuss the basic notions of self-similar groups acting on regular rooted trees and describe a process, that, in analogy to the definition of the Basilica group, produces a new self-similar group having good branching properties from a given one. In the second part, an explicit family of siblings of the Basilica group will be described, with emphasis on the p-congruence subgroup property.

Pro-p groups with quadratic cohomology and generalised pro-p RAAGs

Claudio Quadrelli – University of Milano-Bicocca

Let p be a prime. A pro-p group G has *quadratic* $\mathbb{F}_p$-cohomology if the cohomology graded ring $\bigoplus_{n\geq 0} H^n(G, \mathbb{F}_p)$ of G with coefficients in the finite field $\mathbb{F}_p$, endowed with the graded-commutative cup product, is generated by elements of degree 1, and all its relations are generated by homogeneous relations of degree 2. Such pro-p groups have gained great importance in Galois theory, as by the Rost-Voevodsky Theorem the maximal pro-p quotients of absolute Galois groups — a very mysterious class of profinite groups! — have quadratic $\mathbb{F}_p$-cohomology.

After giving (at a basic level) definition and properties of $\mathbb{F}_p$-cohomology of pro-p groups, we will present the class of *generalised right-angled Artin pro-p group*: such pro-p groups are obtained by "twisting" slightly the definition of pro-p completions of RAAGs, but they yield much more

flexibility. Within this class one may find a wide variety of (new) examples of pro-p groups with quadratic $\mathbb{F}_p$-cohomology, and for these groups the $\mathbb{F}_p$-cohomology ring is computed directly from the associated graph.

This is a joint work with I. Snopce and M. Vannacci.

Hessians, automorphisms of p-groups, and torsion points of elliptic curves

Mima Stanojkovski – MPI MiS Leipzig

We compute the number of automorphisms of p-groups whose commutator structures are determined by Hessian determinantal representations of certain elliptic curves. We will interpret these numbers in terms of the geometry of the defining curves. This is joint work with Christopher Voll.

Groups, graphs, and hypergraphs

Christopher Voll – Universität Bielefeld

One of the fundamental questions one may ask about a finite group is how many conjugacy classes it has. I will report on recent joint work with Tobias Rossmann in which we answer this question uniformly for classes of finite p-groups arising naturally from finite graphs.

I shall explain how understanding conjugacy classes of these p-groups is essentially linked with understanding rank distributions of matrices with prescribed support, and how we attack this problem using methods from toric geometry and p-adic integration.

Pro-p groups with more than 1 end

Thomas Weigel – University of Milano-Bicocca

Following a suggestion of O.V. Melnikov, A.A. Korenev defined in 2004 the *number of ends* of a finitely generated pro-p group G by

$$e(G) = \dim_{\mathbb{F}_p}(H^1(G, \mathbb{F}_p[\![G]\!])) - \dim_{\mathbb{F}_p}(H^0(G, \mathbb{F}_p[\![G]\!])) + 1$$

Thus a finitely generated pro-p group G with more than 1 end, is an infinite pro-p group G satisfying $H^1(G, \mathbb{F}_p[\![G]\!]) \neq 0$. In his paper from 2004, A.A Korenev showed a pro-p analogue of E. Hopf's theorem, i.e., one has $e(G) \in \{0, 1, 2, \infty\}$. Furthermore, $e(G) = 0$ if, and only if, G is finite, and $e(G) = 2$ if and only if, G is infinite and virtually cyclic. The result which is still missing in this context in a pro-p analogue of Stallings' decomposition theorem, i.e., if G is a finitely generated pro-p group with $e(G) = \infty$, then G is either a isomorphic to a free pro-p product $A \amalg_C B$ with amalgamation in a finite subgroup C, or isomorphic to a pro-p HNN-extension $G \simeq \mathrm{HNN}_\beta(A, t)$ where $\beta\colon B \to B_0$ is a homomorphism of finite subgroups B, B_0 of A. In the talk I will give the latest state of the art of this problem.

Virtually free pro-p groups and p-adic representations

Pavel Zalesskii – University of Brasília

We shall discuss the connection between the pro-p group $G = F \rtimes K$, where F is free of rank n and K is finite, and p-adic representations of K in $\mathrm{GL}_n(\mathbb{Z}_p)$.
