# MODELLTHEORIE OBERSEMINAR

## Introduction

The main subject of this semester's Oberseminar is to show a proof of the following theorem:

**Theorem 1** (Johnson, Tran, Walsberg, Ye)**.** *An infinite large stable field is separably closed.*

Currently there are different ways to prove this theorem. Our aim this semester is to follow the first original proof given in [2] which combines some new interesting methods including the so-called *étale-open topology*.

In what follows we give a brief description of the lecture's programme:

## Lecture 1 (October 19): Overview and motivation

This first lecture should introduce the audience to the stable field conjecture and give a brief overview of the content of the seminar. The speaker should briefly recall all the notions from the main theorem (i.e., stable, large and separably closed). In addition, prior results and possible conjectures could also be presented here. The reader should certainly look at the main paper [2], but another source can be these Lectures by D. Marker [3].

## Lectures 2-4: Introduction of algebro-geometric concepts

There will not be a selected speaker for these three lectures. The idea here is to work out together some key exercises to get into grip with some basic concepts of algebraic geometry. The main concepts we need to grasp for each lecture will be given in advance and the participants should come to the lecture having read a definition of such concepts. The first 20-30 minutes we will ask and answer questions around such concepts, and the remaining of the time we will solve together some key exercise about such concepts. The exercises will also be given in advance (so that at least someone has an idea of how to proceed). The references here are mainly [1, 5, 7].

## Lecture 2 (October 26): Sheaves and locally ringed spaces

- Concepts: Sheaves, locally ringed space, stalk, needed categorical concepts (limits, colimits).

- Main exercise: Show that if $A$ is a commutative ring, then $\mathrm{Spec}(A)$ carries the structure of a locally ringed space. For that (rough draft)

- Show that it is enough to check on the base
- Define $\mathcal{O}_{\mathrm{Spec}(A)}(D(f))$ as the localization of $A$ at the multiplicative set of all $g \in A$ with $D(f) \subseteq D(g)$.
- Show that the natural map $A_f \to \mathcal{O}_{\mathrm{Spec}(A)}(D(f)))$ is an isomorphism
- Check the sheaf conditions on the basic open sets $D(f)$.
- Show that the stalks $\mathcal{O}_{\mathrm{Spec}(A),x}$ for $x \in \mathrm{Spec}(A)$ are local rings.

### Lecture 3 (November 2): Schemes and morphisms

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- Concepts: Affine scheme, scheme of finite type, morphism of schemes, $K$-point.

Exercises:
- Give the definitions of $\mathbb{A}^n$, $\mathbb{P}^n$ and $\mathbb{G}_m$. Show that they are reduced schemes of finite type.

### Lecture 4 (November 9): More on morphisms

- Concepts: Base change, smooth morphism, regular points, open (closed) immersions, étale morphism.

Exercises:
- Show that open immersions, closed immersions and smooth morphisms are stable under base change

### Lecture 5 (November 16): Systems of topologies

From this Lecture on, we start again the usual Lecture method we one speaker per Lecture. The main objective of this lecture is go over [2, Section 4].

### 1. Lecture 6 (November 23): Étale open topology

Introduce the étale-open topology and prove that it is a system of topologies. All the material is contained in [2, Section 5].

### 2. Lecture 7 (November 30): Algebra vs topology

Show the main properties of the étale-open topology. In particular, prove [2, Theorem C]. The material is contained in [2, Section 6].

### 3. Lecture 8 (December 7): Stability theory

This lecture is a brief introduction to stability theory with an eye towards stable groups and fields. The speaker should give some of the equivalent definitions of a stable theory (not the order property, number of types, definable types), provide common examples such as algebraically closed fields and separably closed fields. The main reference is Chapter 8 of [6] (it also contains a proof that separably closed fields are stable [6, Example 8.6.7]).

### 4. Lectures 9 and 10 (December 7 and January 11): Stable groups

In this two lectures we introduce stable groups and some of its main properties. The two speakers should prepare these talks in close coordination. The aim is to show that a stable field has a unique additive generic type which also agrees with the multiplicative generic type. Some properties of genericity will be needed. In order to obtain this, some properties on chain conditions and general properties on groups will be needed. The main reference is [4, Chapter 5] which uses tools from different chapters. The lecturer will choose how to order the material to get to the main result [4, Theorem 5.10.], possibly in the next lecture.

### 5. Lecture 10 (January 11): Stable groups 2

Here we continue the study of stable groups. We will also prove a local version of this result which appears in [2, Section 2].

## 6. Final Lecture 11 (January 18): Putting everything together + final comments

Give a proof of the main theorem. The material is contained in [2, Section 9].

## References


[1] Görtz, U., and Wedhorn, T. *Algebraic geometry I. Schemes—with examples and exercises*. Springer Studium Mathematik—Master. Springer Spektrum, Wiesbaden, [2020] ©2020. Second edition [of 2675155]. 1

[2] Johnson, W., Tran, C.-M., Walsberg, E., and Ye, J. The étale-open topology and the stable fields conjecture, 2020. https://arxiv.org/pdf/2009.02319.pdf. 1, 2, 3

[3] Marker, D. Lectures on large stable fields, 2021. http://homepages.math.uic.edu/ marker/mc-S21/. 1

[4] Poizat, B. *Stable groups*, vol. 87 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2001. Translated from the 1987 French original by Moses Gabriel Klein. 2

[5] Scholze, P. Algebraic geometry i, 2016/17. https://www.math.uni-bonn.de/people/ja/alggeoI/notes.pdf. 1

[6] Tent, K., and Ziegler, M. *A course in model theory*, vol. 40 of *Lecture Notes in Logic*. Association for Symbolic Logic, La Jolla, CA; Cambridge University Press, Cambridge, 2012. 2

[7] Vakil, R. The rising sea, 2017. http://math.stanford.edu/ vakil/216blog/FOAGnov1817public.pdf. 1