

Advanced Group Theory Seminar - Summer Semester 2024

Analytic Group Theory

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In this seminar, we delve into different analytic approaches to study groups and their actions. Our journey will be guided by the book [CG], authored by Cohen and Gellander, which serves as our roadmap. According to the authors, this book is an introductory text to the topic, but it can also be used “as a lightweight book to take for a one month trek in the Nepali Himalayas”.

We will mostly focus our attention on amenability, one of the most important properties of analytic group theory. In the last part of the seminar we will introduce property (T). If we have time, we will also look at the Tits alternative.

TALK 1: Amenability

Chapter 2. Cover Sections 1 and 2. In particular, mention Proposition 2.2.3. If time allows, introduce the weak topology, the weak- $*$ topology, and state Theorem 2.2.6 (Banach-Alaoglu Theorem), which would be useful for the next talk.

Keywords: Amenability, Mean, (weak- $*$ topology, Banach-Alaoglu Theorem).

TALK 2: Nets

Chapter 2. Cover Sections 3, 4, and 6. In particular, focus on Proposition 2.3.8, Theorem 2.4.8, and Corollary 2.6.1.

Keywords: Nets, Asymptotically invariant nets, Følner nets, Amenability of abelian groups.

TALK 3: Actions of Amenable Groups

Chapter 3. Cover Sections 1, 2, and 3. Focus on Theorem 3.1.5, Theorem 3.2.1, and Theorem 3.3.1.

Keywords: Compact-convex set, Affine maps, Operations preserving amenability, Elementary amenable groups.

TALK 4: Growth of groups

Chapter 5. Focus on Sections 1 and 3. In particular, mention Corollary 5.3.2. If time allows, present also some results from Section 2.

Keywords: Growth type, Cayley graph, (Growth of nilpotent and solvable groups), Growth of non-amenable groups.

TALK 5: Amenability of topological groups

Chapter 6. Cover Sections 6 and 7. Specifically, Theorem 6.6.8, Theorem 6.7.4, and Corollary 6.7.8.

Keywords: L_p space, Means on topological groups, Lattices in locally compact groups.

TALK 6: Property (T)

Chapter 7. Cover Section 1. Especially, Proposition 7.1.5, Corollary 7.1.7, and Theorem 7.1.10. If time allows, mention Proposition 7.1.8.

Keywords: Unitary representations, Almost invariant vectors, Property (T), Kazhdan pair.

TALK 7 (Optional): The Tits alternative

Chapter 11. The focus of this talk is Theorem 11.0.1 and its proof. Select which sections of the chapter you want to highlight and choose how to present them.

Keywords: Tits alternative.

Main reference

[CG] T. Cohen and T. Gelander *An Invitation to Analytic Group Theory*, 2024. <https://arxiv.org/pdf/2402.15867.pdf>

Further readings

[L] A. Lubotzky, *Discrete Groups, Expanding Graphs and Invariant Measures*, Modern Birkhäuser Classics, Birkhäuser Verlag, Basel, 2010. <https://katalog.ulb.hhu.de/Record/990181523490206441>

If you have problems in getting access to any of the references above, or if you require further details regarding the content or the organization of the talks, please let us know.

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