

# Advanced Seminar on Group Theory – Thompson’s groups

March 8, 2021

In the 1960’s Richard Thompson defined three groups nowadays denoted by  $T$ ,  $F$ , and  $V$  that have remarkable properties, e.g. they are finitely presented, but have unsolvable word problem,  $T$  and  $V$  are simple,  $F$  has property  $FP_\infty$  and is torsion-free. All three groups, as well as numerous generalisations, are still subject to much research.

Our plan is to follow the *Introductory Notes on Richard Thompson’s Groups* [2] by Cannon, Floyd and Parry, and afterwards to look at a selection of more recent results.

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## TALK 1: $F$

Define the group  $F$  and explain what a both a rectangle and tree diagram is. Give examples! At the end, prove Theorem 2.5 in [2]. Source: §1 and §2 in [2].

## TALK 2: More $F$

Prove that  $F$  is finitely presented. Then present some of the results of §4, preferably give a sketch of the proof of Theorem 4.5. Source: §3 and §4 in [2].

## TALK 3: $T$

Define the group  $T$  and prove that it is simple. It is probably necessary to skip some details, but keep some example calculations to give an idea how to work with  $T$ ! Source: §5 in [2].

## TALK 4: $V$

Define the group  $V$ , explain its relationship with  $T$  and  $F$  and define  $V_1$ . Rather than proving that it is simple and isomorphic to  $V$ , present some explicit calculations to prove Lemma 6.1. If there is time, explain the differences and similarities between the proof of simplicity for  $T$  and  $V$ . Source: §6 in [2].

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Time slots 5 and 6 should be filled with two of the following talks (**TALK X** can be chosen twice).

## TALK A: $F$ is of type $FP_\infty$

Explain what it means to be of type  $FP_\infty$ , and follow the proof in [4] why  $F$  is of said type.

## TALK B: Twisted Conjugacy Classes in $F$

A throwback to the topic of the summer semester 2019. Explain what twisted conjugacy classes are, and sketch the proof of Theorem 3.5 in [1].

## TALK C: Cloning Systems

This is a far-reaching generalisation of Thompson’s groups in terms of a limiting process, introduced by Zaremsky and Witzel. Follow along the *User’s Guide to Cloning Systems* [3], that *distill[s] the crucial parts of that 50-page paper into a more digestible form, for those curious to understand the construction but less curious about the gritty details*. Concentrate on your favourites among the many examples.

## TALK X: Your favourite result about Thompson’s groups

Show us whatever pleases you.

## References

- [1] Bleak, Collin; Fel’shtyn, Alexander; Gonçalves, Daciberg L. *Twisted conjugacy classes in R. Thompson’s group F*. Pac. J. Math. 238, No. 1, 1-6 (2008).
- [2] Cannon, J. W.; Floyd, W. J.; Parry, W. R. *Introductory notes on Richard Thompson’s groups*. Enseign. Math., II. Sér. 42, No. 3-4, 215-256 (1996).
- [3] Zaremsky, Matthew C. B. *A user’s guide to cloning systems*. Topol. Proc. 52, 13-33 (2018).
- [4] Zaremsky, Matthew C. B. *A short account of why Thompson’s group F is of type  $F_\infty$* . Topol. Proc. 57, 77-86 (2021).