Advanced Seminar on Group Theory - WISE 2019-2020

Representation Theory of the Symmetric Groups

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For the start of the winter semester group theory seminar we dive into the representation theory of the symmetric groups for a total of five lectures. The first four talks will follow mainly chapter 2 of the book of Sagan [1] and the fifth and last talk follows [2]. The goal of these lectures is to see how one can use purely combinatorial objects, i.e. Young tableaux, to study the representation theory of the symmetric and alternating groups. In particular we will construct for each partition λ of a positive integer n an irreducible $\mathbb{C}S_n$-module S^λ, also known as a Specht module, show that these Specht modules constituent all irreducible $\mathbb{C}S_n$-modules for S_n and see how S^λ decomposes when we restrict or induce it to S_{n-1} or S_{n+1} respectively. In the final talk we will see how Clifford theory can be used to derive the representation theory of the alternating groups from that of the symmetric groups.

Here is a short outline of the talks.

Talk 1: Introduction, overview and basic definitions.
Sketch the procedure for obtaining the irreducible modules for S_n mentioned in the introduction of chapter 2 and cover the content of paragraphs 2.1 and 2.2 (mention definitions 2.2.1 and 2.2.2 only informally) from [1].

Talk 2: Specht modules.
Cover paragraphs 2.3 and 2.4 from [1].

Talk 3: Standard tableaux.
Cover paragraphs 2.5 and 2.6 from [1]. Introduce and illustrate the hook formula (Theorem 3.10.2 of [1]) for computing the number of standard λ-tableaux, i.e. the dimension of S^λ (but skip the proof).

Talk 4: Restricting and inducing S^λ and Young's rule.
Cover paragraph 2.8 from [1]. Furthermore, mention and illustrate Theorem 2.11.2 (Young's rule) from [1], but skip the proof. Also, mention and illustrate Theorem 2.1 (the Murnaghan-Nakayama rule) from [2].

Talk 5: Representation theory of the alternating groups.
Give a short overview, without proofs, of paragraph 3 and mention Lemma 4.1, Theorem 4.2, Lemma 4.3 and Remark 4.4 of [2]. Finally proof Theorem 4.5 of [2].

References

[1] B. Sagan, *The Symmetric Group*, Representations, Combinatorial Algorithms, and Symmetric Functions, second edition, Springer, New York, 2001.

[2] T. Ceccherini-Silberstein, F. Scarabotti and F. Tolli *On the representation theory of the alternating groups*, International Journal of Group Theory, Vol. 2 No. 1 (2013), pp. 187-198. Published online doi: 10.22108/ijgt.2013.2841. http://ijgt.ui.ac.ir/article_2841_21712515b0e5f7db680433e0809ea2ed.pdf