Verbal Width in Groups

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The width $m(w, G)$ of a word w in a group G is the maximal distance to 1 of any element of the Cayley graph of the verbal subgroup $w(G)$ with respect to the natural generating set of all w-values G_w. An open problem is to characterise the words w where for all finite groups G the growth of $m(w, G)$ is bounded in terms of the rank of G. Our goal is to understand the solution of this problem for finite nilpotent groups, as given in the exposition [1].

TALK 1: Generalities and width of lower central words in nilpotent groups.

Define the basic notions of words, width, verbal subgroup, and show first results for lower central words in nilpotent groups.

Plan: Show Corollary 1.2.8, if time permits, give an overview of section 1.3 or 1.4.

TALK 2: Words of infinite width.

In general, no word should have finite width in all groups. But it is necessary to exclude some *silly* examples, about which we will learn here.

Plan: Prove Theorem 3.1.2.

TALK 3: Finitely generated virtually abelian-by-nilpotent groups are verbally elliptic.

A group is called verbally elliptic if every word has finite width. It was proven by Stroud in his thesis that finitely generated virtually abelian-by-nilpotent groups are verbally elliptic.

Plan: Prove Theorem 2.3.1. Explain how this can be improved to Theorem 2.3.6.

TALK 4: Verbal subgroups in profinite groups.

The basics facts on verbal subgroups in profinite groups are explained and profinite analogues of the results of the first talk are given.

Plan: Give the definitions and work your way to prove Theorem 4.1.5.

TALK 5: Pronilpotent groups I.
TALK 6: Pronilpotent groups II.

A $J(p)$-word is a word w such that $w(C_p \wr C_\infty) = 1$. We study the set of $J(p)$-words and their relation to uniform ellipticity of nilpotent p-groups.

Plan: Go through section 4.3 and prove Theorem 4.3.6. Explain necessary results of section 4.2. A natural point for talk I to end would be after Theorem 4.3.3. Coordinate where the first speaker will end.

TALK 7: Words of infinite width in pro-p groups.

Let F be the rank 2 free pro-p group. Then any (non-trivial) word in $F''(F')^p$ has infinite width. This allows to give a characterisation of uniformly elliptic words in finite nilpotent groups.

Plan: Prove Theorem 4.5.1. and 4.5.2.

References

[1] Segal, Dan. *Words: notes on verbal width in groups.* London Mathematical Society Lecture Note Series, 361. Cambridge University Press, Cambridge, 2009.