Advanced Seminar on Group Theory
Winter semester 2023/2024
Representations of reductive p-adic groups

The plan is to learn about the basic set-up (concepts, terminology) for studying smooth representations of reductive p-adic groups. We start with six lectures, based on lecture notes entitled "Representations of reductive p-adic groups" of Fiona Murnaghhan (Toronto), 2009, Sections 1-7:

https://www.math.toronto.edu/murnaghan/courses/mat1197/notes.pdf

The short introduction in Section 1 gives a nice overview and some motivations for much more than we will be able to cover. Section 2 introduces valuations and local fields (mainly the field of p-adic numbers). We suggest that people read these two sections independently and discuss among each other before the first talk.

In a series of six talks we aim to cover Sections 3 to 7 of Murnaghan's notes, leading up to the concept of supercuspidal representations, Theorems of Harish-Chandra and of Jacquet, and the insight that irreducible smooth representations of reductive p-adic groups (over a field of characteristic not equal to p) are admissible. The latter is also the goal of the shorter set of notes by Blondel (2011; link below) which can be consulted in parallel.

Murnaghan's notes are quite complete and contain many examples. Speakers decide themselves which parts they want to cover in detail and which parts they shorten to fit things into their talks. (Of course, it is also possible to add extra material, but please keep in mind that the next speakers will build on previous talks!)

Talk 1: Smooth representations of locally compact totally disconnected groups (Section 3, p. 11-20)

Talk 2: Haar measure, convolution, and characters of admissible representations (Section 4, p. 21-28)

Talk 3: Induced representations - general properties (Section 5, p. 29- 36)

Talk 4: Parabolic induction and Jacquet modules (Section 6, p. 37-49)

Talk 5: Supercuspidal representations and Jacquet's subrepresentation theorem I (Section 7, p.50-55)

Talk 6: Supercuspidal representations and Jacquet's subrepresentation theorem II (Section 7, p.56-62)

(Speakers for Talks 5 and 6 should coordinate with one another where exactly to split the material from Section 7.)

Remark: The notes continue and cover further interesting concepts and results. We can see how things go. If we feel that we want to see more on this topic, we could quite easily extend the series of talks.

Supplementary readings:

(1) Lecture notes of J. Bernstein: "Representations of p-adic groups", 1992

http://www.math.tau.ac.il/~bernstei/Unpublished_texts/unpublished_texts/Bernstein 93new-harv.lect.from-chic.pdf

(2) Lecture notes of C. Blondel: "Basic representation theory of reductive p-adic groups, 2011

https://webusers.imj-prg.fr/~corinne.blondel/Blondel_Beijin.pdf