Singularities, Monodromy and Zeta Functions
Blatt 10

Exercises for discussion in the exercise class on 17.1.2019

Notation: In the following, A always stands for a K-algebra.

Aufgabe 1:

(Leftover from Blatt 9)

Find the type (d, e) of the following filtered modules:

(a) The $K[x_1, x_2]$-module $K[x_1, x_2]/(x_1 x_2)$;

(b) The $K[x, y]$-module $K[x, y]/(y^2 - x^3)$.

Aufgabe 2:

Prove that $\mathcal{G}\mathrm{r}$ is an *exact functor* in the following sense.

Every exact sequence of filtered A-modules

$$0 \to M' \to M \to M'' \to 0$$

which, for every r, induces an exact sequence of K-modules

$$0 \to F_r(M') \to F_r(M) \to F_r(M'') \to 0,$$

gives rise to a exact sequence of *graded* $\mathcal{G}\mathrm{r}(A)$-modules

$$0 \to \mathcal{G}\mathrm{r}(M') \to \mathcal{G}\mathrm{r}(M) \to \mathcal{G}\mathrm{r}(M'') \to 0.$$

Aufgabe 3:

Suppose M is a finitely generated A-module. It was remarked in the last lecture that there is always a *standard filtration* of M, i.e. one such that $\mathcal{G}\mathrm{r}(M)$ is finitely generated. Prove that any standard filtration of M has the same type (d, e).

Aufgabe 4:

(*) Recall **Proposition 2.1.17**: Suppose that K is an algebraically closed field, V a K-vector space, $\dim(V) \leq \aleph_0$, K uncountable and $\phi \in \mathrm{End}_K(V)$. Then there exists $a \in K$ such that $(\phi - a\,\mathrm{id}) \notin \mathrm{Aut}_K(V)$.

Find out if either/both of the cardinality conditions "$\dim(V) \leq \aleph_0$" or "K uncountable" are necessary in the statement.