Singularities, Monodromy and Zeta Functions
Blatt 7

Exercises for discussion in the exercise class on 06.12.2018

Aufgabe 1:

Let $X = \{(x_1, ..., x_n) | f_1(\bar{x}) = ... = f_k(\bar{x}) = 0\}$ be an affine variety over $\mathbb{Z}$ and
$X(R) = \{(x_1, ..., x_n) \in R^n | f_1(\bar{x}) = ... = f_k(\bar{x}) = 0\}$ the set of R-valued points of X.

Show that $X : R \mapsto X(R)$ induces a functor from the category of rings to the category of sets.

Hint: How should one write $X(f)$ for f a ring homomorphism?

Clarification: Here rings are (by definition) commutative and unital; morphisms in the category of rings preserve 1.

Aufgabe 2:

Verify that $X := \{x | v(x) = 1\}$ is a *uniformly definable set* in terms of Definition 1.5.5.
That is, write out X in terms of affine algebraic varieties, $(\mathbb{Z}_p)_{p \in \mathbb{P}}$ and the operations of co-ordinate projection, cartesian product and boolean combination.

Aufgabe 3:

Let $X := \{x | v(x) \geq 0 \wedge \exists y : y^2 = x\}$.

In the lecture we saw that $\mu_p(X(\mathbb{Q}_p)) = \frac{p}{2(p+1)}$ for all $p \geq 3$.

(a) Calculate $\mu_2(X(\mathbb{Q}_2))$.

(b) Write down a single $Z \in \mathcal{C}_0$ such that $\mathrm{ev}_p(Z) = \mu_p(X(\mathbb{Q}_p))$ for every prime p (including 2).

If you're too lazy for (a), you can denote the rational number $\mu_2(X(\mathbb{Q}_2))$ by r and solve (b) with that.