Mathematisches Institut
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WS 2018/19
18.10.2018
Sheet 1

Stochastic Processes and Stochastic Analysis II
Exercise sheet

Please gather in groups of 2 people to hand in your answers.

Exercise 1: *(6 points)* Let $(B_t)_{t\geq 0}$ be a one-dimensional Brownian motion. Compute $\langle Z \rangle_t$ for $Z_t := B_t^2 - t$.

Exercise 2: *(6 points)* Let $(B_t)_{t\geq 0}$ be a one-dimensional Brownian motion. Show that

$$\langle Z \rangle_t = \int_0^t Z_s^2 f^2(s) ds \quad a.s.$$

for $Z_t := \exp\left(\int_0^t f(s) dB_s - \frac{1}{2}\int_0^t f^2(s) ds\right)$, where $f : \mathbb{R}_+ \to \mathbb{R}$ is a continuous function.

Exercise 3: *(8 points)* Let $(B_t)_{t\geq 0}$ be a one-dimensional Brownian motion. Let $Y \in L^2$ be the a.s. unique solution of

$$dY_t = r dt + aY_t dB_t, \quad Y_0 = 1$$

for some constants $r, a \in \mathbb{R}$.

(a) Compute $\mathbb{E}[Y_t]$ and $\mathbb{E}[Y_t^2]$ explicitly.
(b) Let $Z_t = \exp(-aB_t + \frac{1}{2}a^2 t)$. Calculate dA_t for $A_t = Y_t Z_t$.
(c) Determine the solution Y_t explicitly as a function of B_t.

Hand in on: Thursday, 25.10.2018
Revision: Tuesday, 30.10.2018 in Exercise lesson