Mathematisches Institut
der Heinrich-Heine-Universität
Düsseldorf
Jun.-Prof. Dr. C. Rojas-Molina / Dr. E. Sönmez

WS 2018/19
10.01.2019
Sheet 10

Stochastic Processes and Stochastic Analysis II
Exercise sheet

Please gather in groups of 2 people to hand in your answers.

Exercise 28: *(6 points)* Let $X = (X_t)_{t\geq 0}$ be a Markov chain with state space $S = \{0, 1\}$ and Q-matrix given by

$$Q = \begin{pmatrix} -\beta & \beta \\ \delta & -\delta \end{pmatrix}$$

for $\beta, \delta > 0$. Assume that $X_0 = 0$ and define $T = \inf\{t \geq 0 : X_t = 1\}$. Show that

$$T \overset{\mathrm{d}}{=} \sum_{i=1}^{G} \tau_i,$$

where $G, \tau_1, \tau_2, \ldots$ are independent random variables, τ_i is exponentially distributed with parameter 1 for every i and G has a geometric distribution with parameter β.

Exercise 29: *(8 points)* Let $X = (X_n)_{n\in\mathbb{N}}$ be a discrete-time Markov chain with state space S and $N = (N_t)_{t\geq 0}$ a Poisson process with intensity 1 independent of X. For $t \geq 0$, $x, y \in S$ define

$$p_t(x, y) = P_x(X_{N_t} = y).$$

Show that

$$\sum_{z\in S} p_t(x, z)p_s(z, y) = p_{t+s}(x, y)$$

for all $t \geq 0$, $x, y \in S$.

Exercise 30: *(6 points)* Let $X = (X_t)_{t\geq 0}$ be a Markov chain with Q-matrix Q and jump times $(J_n)_{n\in\mathbb{N}}$. Define $T_i = \inf\{t \geq J_1 : X_t = i\}$ for every state i. Show the following:

(a) If $q_i = 0$ or $P_i(T_i < \infty) = 1$ then i is recurrent and $\int_0^\infty p_{ii}(t)dt = \infty$.

(b) If $q_i > 0$ and $P_i(T_i < \infty) < 1$ then i is transient and $\int_0^\infty p_{ii}(t)dt < \infty$.

Hand in on: Thursday, 17.01.2019
Revision: Tuesday, 22.01.2019 in Exercise lesson