Mathematisches Institut
der Heinrich-Heine-Universität
Düsseldorf
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WS 2018/19
17.01.2019
Sheet 11

Stochastic Processes and Stochastic Analysis II
Exercise sheet

Please gather in groups of 2 people to hand in your answers.

Exercise 31: *(6 points)* Let $X = (X_t)_{t\geq 0}$ be a Feller process on $\mathbb{R}$ with probability semigroup given by

$$P_t f(x) = f(x+t)$$

for $f \in C_0(\mathbb{R})$. Show that the corresponding generator of $(P_t)_{t\geq 0}$ is given by $\mathcal{L}f = f'$ with

$$\mathcal{D}(\mathcal{L}) = \{f \in C_0(S) : f' \in C_0(S)\}.$$

Exercise 32: *(8 points)* Consider $B = (B_t)_{t\geq 0}$ a one-dimensional Brownian motion with probability semigroup given by

$$P_t f(x) = \int_{-\infty}^{\infty} p_t(x,y) f(y) dy,$$

where $p_t(x,\cdot)$ is the density of the $\mathcal{N}(x,t)$ distribution. One can show that the operator $\mathcal{L}f = \frac{1}{2}f''$ with domain

$$\mathcal{D}(\mathcal{L}) = \{f \in C_0(S) : f', f'' \in C_0(S)\}$$

is the generator of $(P_t)_{t\geq 0}$. Prove that for every $f \in \mathcal{D}(\mathcal{L})$

$$\mathcal{L}f = \lim_{t\downarrow 0} \frac{P_t f - f}{t}.$$

Exercise 33: *(6 points)* Show that there is no probability generator whose restriction to smooth functions is given by $\mathcal{L}f = f'''$.

Hand in on: Thursday, 24.01.2019
Revision: Tuesday, 29.01.2019 in Exercise lesson