Mathematisches Institut
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WS 2018/19
29.11.2018
Sheet 7

Stochastic Processes and Stochastic Analysis II
Exercise sheet

Please gather in groups of 2 people to hand in your answers.

Exercise 19: *(6 points)* Let $(X_n)_{n\in\mathbb{N}}$ be the simple random walk on $\mathbb{Z}$ started in $X_0 = 0$ with transition probabilities given by

$$p_{ij} = \begin{cases} p, & j = i+1 \\ q, & j = i-1 \\ 0, & \text{else,} \end{cases}$$

where $p, q \geq 0$ with $p + q = 1$. Show that $(X_n)_{n\in\mathbb{N}}$ is recurrent if $p = q = \frac{1}{2}$ and transient otherwise.
Hint: Use the Stirling formula:

$$n! \sim \sqrt{2\pi n}(\frac{n}{e})^n,\ n \to \infty.$$

Exercise 20: *(8 points)* Let $(X_n)_{n\in\mathbb{N}}$ be the simple symmetric random walk on $\mathbb{Z}^3$ started in $X_0 = 0$ with transition probabilities given by

$$p_{ij} = \begin{cases} \frac{1}{6}, & |i-j| = 1 \\ 0, & \text{else.} \end{cases}$$

Show that $(X_n)_{n\in\mathbb{N}}$ is transient by proceeding as follows:

(a) Determine an expression for $p_{00}^{(2n)}$.

(b) Use the Stirling formula and give an asymptotic upper bound for $p_{00}^{(2n)}$ in (a) in the case $n = 3m$ for some $m \in \mathbb{N}$.

(c) Conclude that

$$\sum_{n=0}^{\infty} p_{00}^{(n)} < \infty.$$

Exercise 21: *(6 points)* Prove the following statements:

(a) Let C be a communicating class. Then either all states in C are recurrent or all are transient.

(b) Every recurrent class is closed.

(c) Every finite closed class is recurrent.

Hand in on: Thursday, 06.12.2018
Revision: Tuesday, 11.12.2018 in Exercise lesson