Mathematisches Institut
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WS 2018/19
06.12.2018
Sheet 8

Stochastic Processes and Stochastic Analysis II
Exercise sheet

Please gather in groups of 2 people to hand in your answers.

Exercise 22: *(8 points)* Let $(X_n)_{n\in\mathbb{N}}$ be a Markov chain on the finite state space S and assume that $A \subset S$ is absorbing. For $i \in S$ define $h_i := P_i(T < \infty)$, where

$$T := \inf\{n \geq 0 : X_n \in A\}.$$

Show that $M_n := h(X_n)$, $n \in \mathbb{N}$, is a martingale with respect to the canonical filtration.

Exercise 23: *(6 points)* Let $(X_n)_{n\in\mathbb{N}}$ be the simple symmetric random walk on $\mathbb{Z}$ started in $X_0 = 0$ and $a, b \in \mathbb{N}$.
(a) Compute $P(X_n \text{ hits } -a \text{ before } b)$.
(b) Compute $\mathbb{E}[T]$ for $T := \inf\{n \geq 0 : X_n \in \{-a, b\}\}$.

Exercise 24: *(6 points)* Let $X = (X_n)_{n\in\mathbb{N}}$ be a Markov chain with transition matrix P on the state space S. Write $S = D \cup \partial D$ for some set D, let T be the hitting time of ∂D for X starting in D and suppose that c and f are non-negative functions. Show that $\phi^{(n)}$, $n \in \mathbb{N}$, defined by

$$\phi_i^{(n)} = \mathbb{E}_i\Big[\sum_{k=0}^{n} c(X_k)1_{\{k<T\}} + f(X_T)1_{\{T\leq n\}}\Big]$$

for every $i \in S$ satisfies

$$\begin{cases} \phi^{(n+1)} &= c + P\phi^{(n)} \quad \text{on } D \\ \phi^{(n+1)} &= f \qquad\qquad \text{on } \partial D. \end{cases}$$

Hand in on: Thursday, 13.12.2018
Revision: Tuesday, 18.12.2018 in Exercise lesson