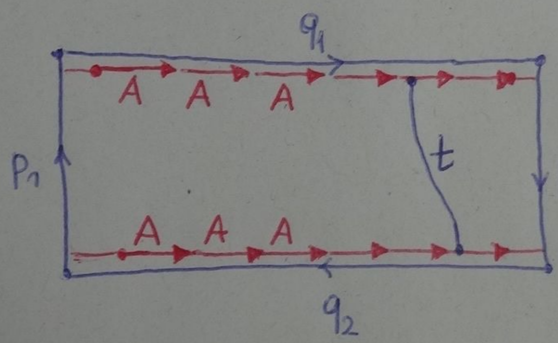
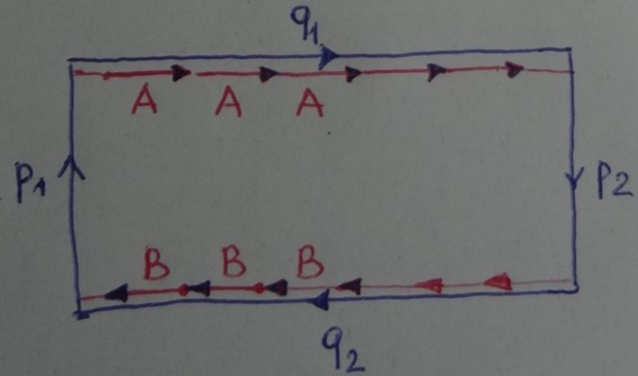
About the role of oddness of exponent n

Lem. 18.6 Let Δ be a reduced circular diagramm of rank i with contour $p_1 q_1 p_2 q_2$, where $\varphi(q_1)$ and $\varphi(q_2^{-1})$ are periodic words with period A simple in rank i.
If $|p_1|, |p_2| < \alpha \cdot |A|$ and $|q_1|, |q_2| > (\frac{5}{6} h + 1) \cdot |A|$, then q_1 and q_2 are A-compatible in Δ

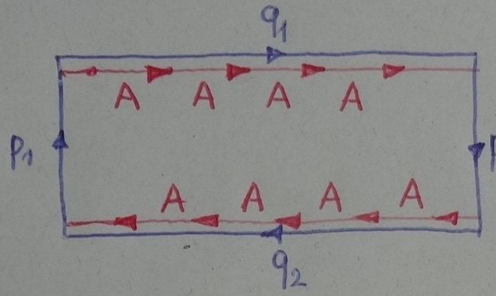


$\boxed{t \underset{G(i)}{=} 1}$

Lem. 18.8 — " —, where $\varphi(q_1)$ and $\varphi(q_2)$ are periodic words with periods A and B simple in rank i, $|A| \geq |B|$.
If $|p_1|, |p_2| < \alpha \cdot |A|$ and $|q_1| > \frac{3}{4} h \cdot |A|$, $|q_2| > h \cdot |B|$, then $A \underset{G(i)}{\sim} B^{\pm 1}$. Moreover, if $\varphi(q_1)$ and $\varphi(q_2^{-1})$ begin with A and B^{-1}, then $\boxed{A \underset{G(i)}{=} \varphi(p_1)^{-1} B^{\pm 1} \varphi(p_1)}$.



Lem. 18.9 Let Δ be a reduced circular diagram of rank i with contour $p_1 q_1 p_2 q_2$, where $\varphi(q_1)$ and $\varphi(q_2)$ are periodic words with period A simple in rank i.
If $|p_1|, |p_2| < \alpha \cdot |A|$, then $|q_1|, |q_2| \le h \cdot |A|$.



Proof. By Lem. 19.4 and 19.5 Δ is an A-map with smooth q_1, q_2.

We assume (to the contrary) $|q_1| \geq |q_2|$; $|q_1| > h \cdot |A|$.

For simplicity, we assume that q_1 starts with A and q_2 ends with A.

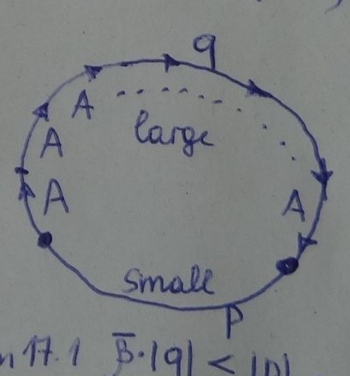
By Thm. 17.1, $|q_2| > \bar{\beta} |q_1| - |p_1| - |p_2| > (\bar{\beta} h - 2\alpha)|A| > \frac{3}{4} h |A|$

By Lem. 18.8, $A \underset{G(i)}{=} Z^{-1} A^{\pm 1} Z$, where $Z = \varphi(p_1)$

Case 1 $A \overset{G(i)}{=} Z^{-1} A Z$. Consider p_2 and $q_2 p_1 q_1$:

$\varphi(p_2) \overset{(i)}{=} \tilde{A} \cdot A^{\ell} \cdot Z A^{k} \cdot \tilde{\tilde{A}}$, where $\tilde{A}, \tilde{\tilde{A}}$ are subwords of A; $\ell, k > 0$.

$\underbrace{\tilde{A}^{-1} \varphi(p_2) \tilde{\tilde{A}}^{-1} Z^{-1}}_{\text{small: } < 2|A| + 2\alpha|A|} \overset{(i)}{=} \underbrace{A^{\ell+k}}_{\text{large: } > k|A| \gtrsim h|A|}$



Δ'

Δ' is a new diagram with large smooth q and small p. By Thm 17.1 $\bar{\beta} \cdot |q| < |p|$. Contrad.

Case 2. $A \overset{G(i)}{=} Z^{-1} A^{-1} Z$. Then $Z^{-2} A Z^{2} = A$, hence $\forall m$

$$A^m \overset{G(i)}{=} Z^{-2} A^m Z^2 .$$

By Lem. 18.7 $\Rightarrow Z^2 \overset{G(i)}{=} A^d$ for some d

Lem 18.7 Let $Z_1 A^{m_1} Z_2 = A^{m_2}$, $m = \min(m_1, m_2)$, where A is simple in rank i. If $\lvert Z_1\rvert + \lvert Z_2\rvert < (\gamma(m - \frac{5}{6}h - 1) - 1)\lvert A\rvert$, then Z_1 and Z_2 are equal in rank i to powers of A.

Thus, $Z^2 \overset{G(i)}{=} A^d$, $A^d \overset{G(i)}{=} Z^{-1} A^{-d} Z$

$$\Rightarrow \quad A^{2d} \overset{G(i)}{=} 1 \quad \overset{\text{Lem } 18.3}{\Longrightarrow} \quad d = 0 . \quad (\text{since } A \text{ is simple})$$

Lem 18.3 If $X \overset{G(i)}{\neq} 1$ and X has finite order in rank i, then it is conjugate in rank i to a power of some period of rank $k \leq i$

$\Rightarrow Z^2 \overset{G(i)}{=} 1$. If $Z \overset{G(i)}{\neq} 1$, then by Lem 18.3.

$Z \underset{G(i)}{\sim} B^e$, where B is a period of $rk \leq i$

But $n = Ord(B)$ is odd!

Thus, $Z \overset{G(i)}{=} 1 \Rightarrow A^2 \overset{G(i)}{=} 1$

Analogously $A \overset{G(i)}{=} 1$. Contradicts to simplicity of A.