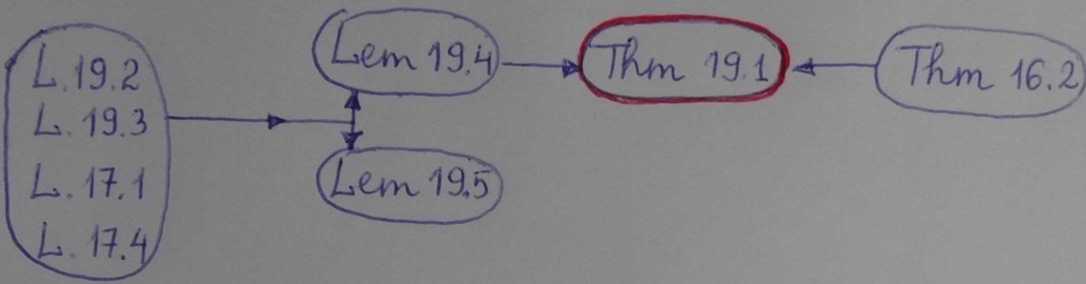
Thm 19.1 The law $X^n = 1$ holds in the group $G(\infty) = B(\mathcal{A}, n)$ and if $|\mathcal{A}| > 1$, $n > 10^{10}$ odd, then $G(\infty)$ is infinite.



Lem. 19.4. A reduced diagram Δ of rank $i+1$ is an A-map.

Δ is a diagram over $G(i+1) = \langle \mathcal{A} \mid \mathcal{X}_1^n \cup \cdots \cup \mathcal{X}_{i+1}^n \rangle$,

$$\widetilde{\mathcal{X}}_{i+1} = \{ w \in F(\mathcal{A}) \mid |w| = i+1,$$

$$w \underset{G(i)}{\not\sim} B^k \quad (1 \le k < n),\ B \in \mathcal{X}_1 \cup \cdots \cup \mathcal{X}_i,$$

$$w \underset{G(i)}{\not\sim} w' \quad \text{if } |w'| < |w| \}$$

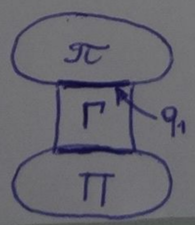
$\mathcal{X}_{i+1} \subseteq \widetilde{\mathcal{X}}_{i+1}$ is a maximal subset with the property:

if $A \not\equiv B \in \mathcal{X}_{i+1}$, then $A \underset{G(i)}{\not\sim} B^{\pm 1}$.

Def A map Δ is called an A-map if

1) For each cell Π of rank j we have $|\partial \Pi| \geq n \cdot j$

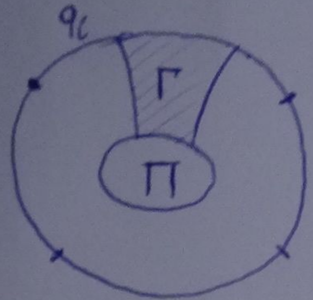
2) For any subpath $q \subseteq \partial \Pi$, where $rk(\Pi) = j$, we have:
If $|q| \le \max\{j, 2\}$, then q is geodesic.

3) 

~~If (π, Γ, Π)~~

If $(\Pi, \Gamma, \pi) \geq \varepsilon$, then $|q_1| < (1+\gamma) \cdot rk(\Pi)$

Theorem 16.2 Let Δ be an A-map of non-zero rank with $1 \le l \le 4$ sections of $\partial\Delta$. Then there exists an $\mathcal{R}$-cell $\Pi \subseteq \Delta$ and a contiguity submap Γ of Π to a section q_i of $\partial\Delta$, s.t.



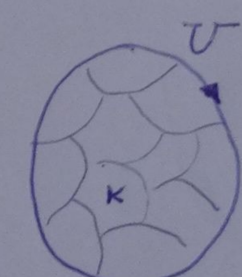
$$\mathrm{rk}(\Gamma)=0 \text{ and } (\Pi,\Gamma,q) \geq \varepsilon .$$

Thm. 19.1. The law $X^n=1$ holds in the group $G(\infty)=B(\mathcal{A},n)$ and if $|\mathcal{A}|>1$, $n>10^{10}$ odd, then $G(\infty)$ is infinite.

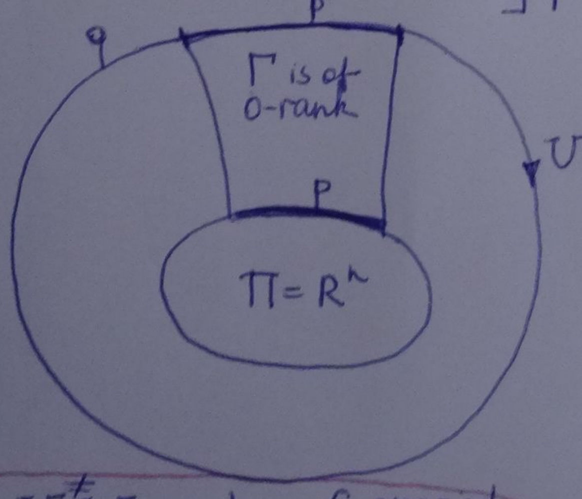
Proof Suppose $1 \neq V \in F(\mathcal{A})$ is a cyclically reduced word, s.t $V=1$ in $G(\infty)$. Then $\exists\, \kappa$, s.t. $V=1$ in $G(\kappa)$.

Then there exists a diagram Δ over $G(\kappa)$ with $\varphi(\partial\Delta)=V$.

By 19.4., Δ is an A-map



By Thm. 16.2. $\exists\, \Pi = R^n$
$\exists\, \Gamma$ as above



Since $(\Pi,\Gamma,q) \geq \varepsilon$,

$|p| \geq \varepsilon \cdot |\partial\Pi| = \varepsilon \cdot n \cdot |R| \geq 15|R|$.

$\Rightarrow$ p contains 13 times R.

$p' \underset{F(\mathcal{A})}{=} p$ contains R^{13}.

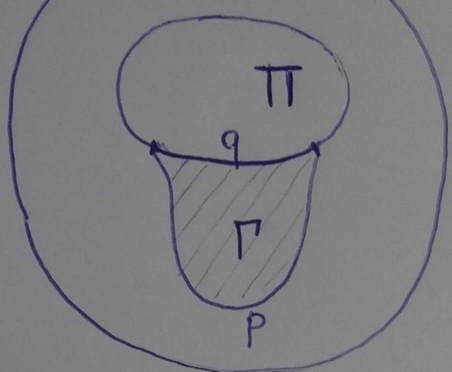
Lem. If $V \neq W$ are two 6-aperiodic words in $F(\mathcal{A})$, then $V \underset{G(\infty)}{\neq} W$.

Proof Otherwise $\exists\, \kappa$: $VW^{-1} \underset{G(\kappa)}{=} 1$ for some κ. $\Rightarrow V'(W')^{-1} \underset{G(\kappa)}{=} 1 \Rightarrow$ V' or W' contains R^6.

<u>Lem 19.4</u> A reduced diagram Δ of rank $i+1$ is an A-map

<u>Proof</u> 1) is clear.

2)



$rk(\Pi) = j \leq i+1$, $q \subseteq \partial\Pi$,

$|q| \leq \max(j, 2)$.

Suppose $|p| < |q|$

$\boxed{\text{Case 1}}$ $\Pi \not\subseteq \Gamma$.

Then, by induction, Γ is an A-map.

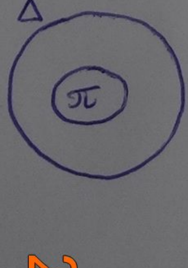
(Subcase 1) Suppose $|q| \leq 2$.

Then $|\partial\Gamma| = |q| + |p| \leq 2 + 1 = 3$

$\xRightarrow{C.17.1} rk(\Gamma) = 0.$

$\boxed{\text{Cor. 17.1}}$ If a circular A-map Δ contains an $\mathcal{R}$-cell π, then

$|\partial\Delta| > \bar{\beta}\, |\partial\pi|$ [redacted]

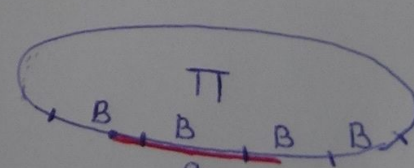


In our situation, for any $\mathcal{R}$-cell $\pi \subseteq \Gamma$, we have

$3 = |\partial\Gamma| > \bar{\beta}\, |\partial\pi| = \bar{\beta} \cdot rk(\pi) \cdot n \quad \Rightarrow rk(\pi) = 0. \Rightarrow rk(\Gamma) = 0.$

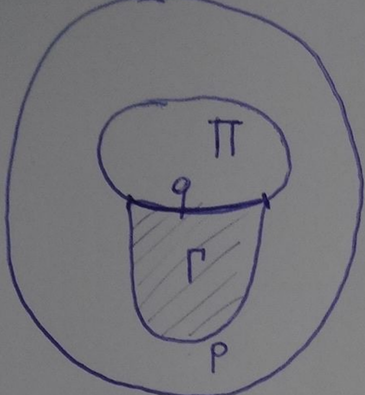
$\Rightarrow \varphi(p) \underset{F(A)}{=} \varphi(q)$ ← ~~cyclically~~ reduced

$|p| < |q|$



each B is cyclically reduced

A contradiction.



$rk(\Pi) = j \le i+1$, $q \subseteq \partial \Pi$,

$|q| \le \max(j, 2)$

Suppose $|p| < |q|$

$\boxed{\text{Case 1}}$ $\Pi \subseteq \Gamma$.

Then, by induction Γ is an A-map.

(Subcase 2) Suppose $|q| \le j$

Then $|\partial \Gamma| = |q| + |p| \le j + (j-1) < 2j$.

$$\xRightarrow{C.17.1} \quad rk(\Gamma) < j.$$

$\boxed{\text{Cor. 17.1}}$ If a circular A-map Δ contains an $\mathcal{R}$-cell π, then

$$|\partial \Delta| > \bar{\beta} \cdot |\partial \pi|$$

In our situation, for any $\mathcal{R}$-cell $\pi \subseteq \Gamma$, we have

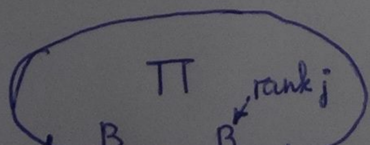
$$2j > |\partial \Gamma| > \bar{\beta} \cdot |\partial \pi| = \bar{\beta} \cdot rk(\pi) \cdot n$$

$$\Rightarrow \quad rk(\pi) < \frac{2j}{\bar{\beta} \cdot n} < j \quad \Rightarrow rk(\Gamma) = \max_{\pi \subseteq \Gamma} rk(\pi) < j.$$

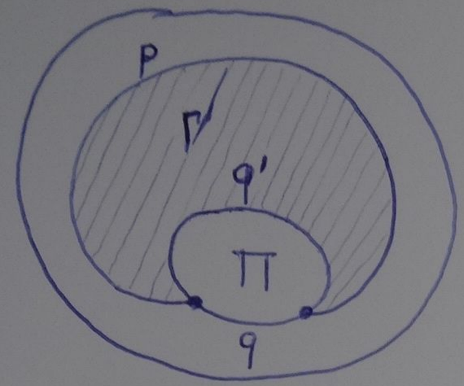
$$\Rightarrow \varphi(q) \underset{G(j-1)}{=} \varphi(p)$$

$$|q| > |p|$$

A contradiction:



(Since $|q| \le j = |B|$, some $qq' \sim B \Rightarrow pq' \underset{G(j-1)}{\sim} B$)



$rk(\Pi) = j \leq i+1$, $q \subseteq \partial\Pi$

$|q| \leq \max(j, 2)$

Suppose $|p| < |q|$

$\boxed{\text{Case 2}}$ $\Pi \not\subseteq \Gamma$

Again, by induction, Γ is an A-map

<u>Remark</u> No cell of Γ is A-compatible with q', where $\partial\Pi = A^n$
Otherwise some cell $\pi \subseteq \Gamma$ is A-compatible with Π, and Δ were not reduced.

$\overset{L.9.5}{\Longrightarrow}$ q' is a smooth section of $\partial\Gamma$

<u>Lem 9.5</u> Let q' be a section of the countor of a reduced diagram Γ of some rank l.
Suppose $Lab(q')$ is A-periodic, where one of the following holds

1) A is simple in rank l
2) A is a period of rank $j \leq l$ and Γ has no cells of rank j A-compatible with q.

Then q' is a smooth section of $\partial\Gamma$. (even $|A|$-smooth)

$\overset{T.17.1}{\Longrightarrow}$ $\bar{\beta} \cdot |q'| \leq |p|$. Observe $|q'| \geq |\partial\Pi| - |q| \geq |\partial\Pi| - j = |\partial\Pi| - |A| = |\partial\Pi| - \frac{|\partial\Pi|}{n}$.

$$\Rightarrow |q| \leq j = \frac{|\partial\Pi|}{n} \leq \frac{|q'| \cdot \frac{n}{n-1}}{n} = \frac{|q'|}{(n-1)} \leq \frac{\bar{\beta}}{(n-1)} \cdot |p| < |p|. \Rightarrow q \text{ is geodesic.}$$