Thm 19.4 (1) Every element of $G = \mathcal{B}(\mathcal{A}, n)$ is conjugate to a power of a period of some rank i.

Proof. Since $G \models \forall z\ (z^n = 1)$, given $X \neq 1$ one finds i s.t. $X^n \overset{i}{=} 1$.
Lemma 18.3 says that X is conjugate (in rank i) to a power of some period of rank $k \le i$.

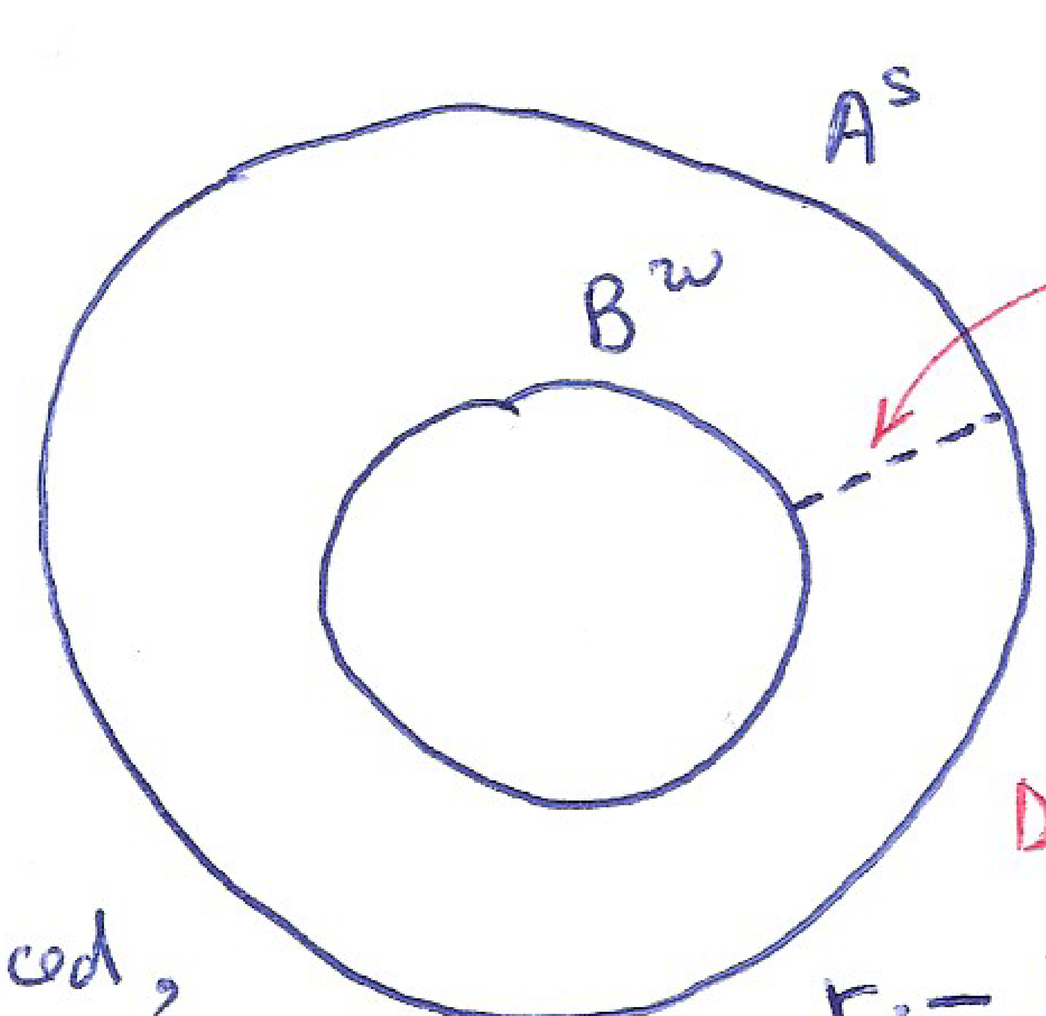
Thm 19.4 (2) A period of any rank has order n.

Proof. Let $m < n$ and $A^m \overset{i}{=} 1$.
period of rank $\le i$

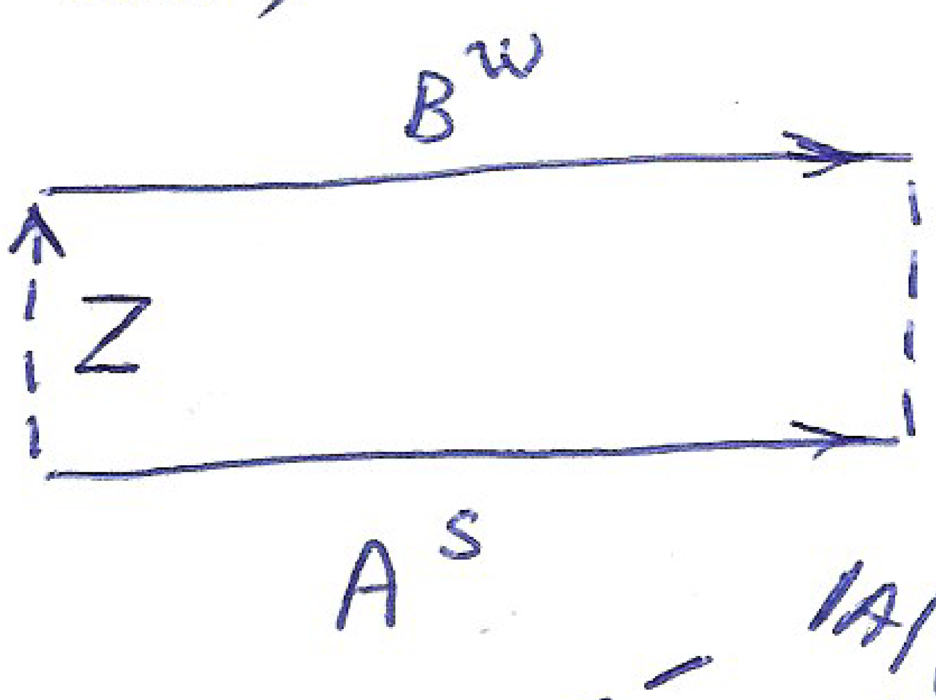
A reduced diagram Δ of this equation is an A-map (L. 19.4). By L. 13.3 we may assume Δ has no cells of $rk(A)$ A-compatible with $\partial\Delta = q$. By L. 19.5 q is smooth – contradiction with 17.1

Thm 19.4 (3) A, B = periods of rk i, j and $G \models A^k \sim B^l$. Then $A = B$ and $A^k = B^l$.

Proof. Assume $j \le i$, $|\langle A^k\rangle| = m$. Replace k by s and l by w s.t. $s | n$, $0 < s \le \frac{n}{3}$ and $|w| \le \frac{n}{2}$
(n is odd)



L. 17.1: $< \gamma(|A^s| + |B^w|)$



$|\partial\Delta| \le (|A^s| + |B^w|)(1+\gamma) \le |A|(|s| + |w|)(1+\gamma) \le \frac{5n}{6}$

Def. A-map + Cor 17.1

$r := r(\Delta) < \frac{|\partial\Delta|}{n\,\bar\beta} < |A| = i$

- A is simple in r. r, i.e. A^{st} is smooth in
L. 19.3(2)

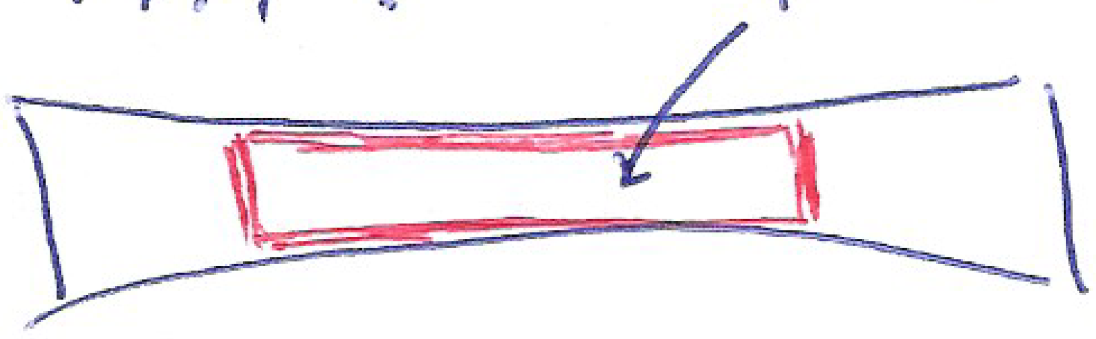
a reduced diagram for $\underline{A^{st} Z B^{-wt} Z^{-1} \overset{r}{=} 1}$

B^{wt} is also smooth ($r < j \leftarrow$ apply L.19.3(2) + 19.5)

($j \leq r \leftarrow$ apply L. 13.3 + L.19.5 for periods)
up to $n \cdot n_1$

Apply L.19.2 ($|q_2| \geq \varepsilon n |B|$, $|q_1| \geq (1+\gamma)|A|$,
$\max(|p_1|, |p_2|) < 5n \cdot \min(|A|, |B|) \Rightarrow A \sim_r B^{\pm 1}$),
but we need that A and B are simple in $\underline{r}$ and
(for 19.3(3) } $r < \min(i,j)$
i.e. $A = B$

To have this take large t and apply L.17.5 and 17.4 : r' of $r . r_1 < j$



By L.18.7 $Z \overset{r_1}{=} A^d$,
i.e. $A^k = B^l$.

Thm 19.4 (4) If $\langle X \rangle \cap \langle Y \rangle \neq 1$, then $\exists Z$ $\langle X \rangle \subseteq \langle Z \rangle$ and $\langle Y \rangle \leq \langle Z \rangle$

By Th.19.4 (1) may assume $X = A^k$, $Y = Z B^l Z^{-1}$ for periods A, B of r. $i \geq j$, $X^a = Y^b \neq 1$

By Thm.19.4 (3) $A = B$. Find s, t : $s | n$,
$|s| \leq n/3$, $|t| \leq \frac{n}{2}$, $1 \neq A^s = Z A^t Z^{-1}$

By the argument of Thm 19.4 (3) $A^s \overset{r}{=} Z A^t Z^{-1}$
$r < i$

and by L. 18.7 , $Z = A^d$, i.e. $X, Y \in \langle A \rangle$

Thm 19.5 $\forall X \in G$ $C(X)$ is cyclic of order $\underline{n}$.

Proof. Thm 19.4 (1) , 19.2 (abelian $\Rightarrow$ cyclic) , 19.4 (4) $\square$

Thm 19.6 Any finite sbgrp of $\mathcal{B}(A,n)$ is cyclic

(By 19.2 and 19.4)

Thm 19.7 $\mathcal{B}(A,n)$ is free for $x^n = 1$,

i.e. for any k-generated $< g_1 \dots g_k \mid \mathcal{R} >$

and map $a_i \to g_i$ $i \leq k$ extends

to a homom. $\mathcal{B}(A,n) \to < g_1 \dots g_k \mid \mathcal{R} >$

($|A| = k$)

(By Thm 19.1.) + argument of 19.1

Thm 19.3 The set of relations of $\mathcal{B}(A,n)$ is infinite, indep. and does not follow from any finite set of relations.