Description of the joint Düsseldorf – Gliwice online research seminar
Geometric group theory (October 2020 – February 2020)
(Organizers: Oleg Bogopolski, Alexander Iwanow, Agnieszka Bier)

Aim. We read the famous book of Ol'shanskii "Geometry of defining relations in groups" to learn the technique of graded diagrams, which enabled to obtain a number of sophisticated results in group theory. We look at several papers around it to see how this technique works.

Organisation. There will be no usual talks. According to the plan presented below, everybody reads a piece of the book assigned to a current week and writes questions and comments to the moderator of this semester (Oleg Bogopolski). The moderator collects all questions, publishes it on the webpage of the course, and assigns a meeting, where the questions will be discussed and clarified.

Plan.

1) 29 October.

   We meet to see each other, to discuss the organisation and to take in mind some motivating questions. Oleg Bogopolski will present some beautiful results obtained with the help of Ol'shanskii's technique.

2) 29 October – 5 November.

   We read Introduction and Chapter 1 (General concepts of group theory). Students take a quick look at §1 – §3. Everybody read §4. A special attention is expected for §4.3 (Words and subwords).

3) 5 November – 12 November.

   We skip Chapter 2 (Main types of groups and subgroups) and Chapter 3 (Topological spaces). We read the first two paragraphs of Chapter 4 (Diagrams over groups):

   §11. Visual interpretation of the deduction of consequences of defining relations (pages 112-125).

   §12. Small cancellation theory (pages 126-136).

4) 12 November – 19 November.

   We read the last paragraph of Chapter 4.
   §13. Graded diagrams (pages 137-151).

5) 19 November – 26 November.

   We begin to read Chapter 5 (A-maps). Be ready to suffer :) – this is a technical chapter.
   §14 Contiguity submaps (pages 152-164).

6) 26 November.

   Oleg Bogopolski explains the Rips construction:

   E. Rips, *Subgroups of small cancellation groups*,
   Bull. London Math. Soc., **14** (1982), 45-47.

7) 26 November – 3 December.

We continue to read §14 "Contiguity submaps" (pages 152-164).
This paragraph introduces some geometrical notions and is crucial.
The moderator shows pictures.

8) 3 December – 10 December.

We read §15 "Conditions on the grading" (pages 164-173).
This small paragraph contains first computations.

9) 10 December – 17 December.

We continue to read §15 "Conditions on the grading" (pages 164-173).
The aim is to understand all theorems in this paragraph without long computations.

10) 17 December – 24 December.

We read only formulations in §16 "Exterior arcs and γ-cells'" (pages 174-181).
Pay attention to Corollary 16.1

11) 24 December.

Informal meeting.

12) 24 December – 7 January.

We read the first two parts of §17 "Paths that are nearly geodesic and cuts on A-maps" (pages 181-185). The aim is to see how lemmas from §15 work. [Oleg and Sasha]
During the 2-week-vocations, please don't forget to send me questions.

13) 7 January – 14 January.

We continue to read the second and the third parts of §17 "Paths that are nearly geodesic and cuts on A-maps" (pages 183-189) [Sasha and Savelii]. The aim is to see how lemmas from §15 work.

14) 14 January – 21 January.

We start to read Chapter 6 "Relations in periodic groups" (pages 194-213).
The aim is to understand formulations and relationships between lemmas leading to the proof of the main Theorem 19.1. This theorem says that the free Burnside group $B(m, n)$ with $m > 1$ generators and of exponent n (which is an arbitrary sufficiently large odd number) is infinite.

15) 21 January – 28 January.

Oleg will present the proofs of Lemmas 19.4 and 19.5, and of the main Theorem 19.1.
If time will remain, Sasha will explain the structure of the proof of Lemma 18.6.

16) 28 January – 4 February.

Sasha, Saveliy, and Oleg will explain the proof of Lemma 18.6.

17) 4 February – 11 February.

We discuss the remaining parts of §18.

18) 11 February – 18 February.

I think that we will not read the rest of this book. Perhaps we will switch to some papers which use Ol'shanskii's method. In any case we will discuss our further plans.