Chapter 6

Defs. of simple words, presentation of B(A, n)

Lemma 18.1: $X \overset{i}{\sim} Y^m$, Y - period of rank $j \leqslant i$ or simple in rank i

<u>Induction on rank i</u>

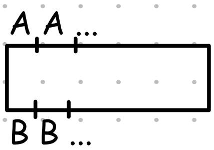
Rd. dgrs. of rank i are A-maps, if q is B-periodic & no comp. cells, then it's smooth of rank |B|

Lemmas 18.2-18.4

Lemma 18.5: $X \overset{i}{\sim} Y \Rightarrow X = ZYZ^{-1}$, $|Z| \leqslant \bar{\alpha}(|X|+|Y|)$

<u>Induction on |A| + |B|</u>

Lemmas 18.6-18.8



Lemma 18.9 (now n is odd)

Lemmas 19.1-19.3

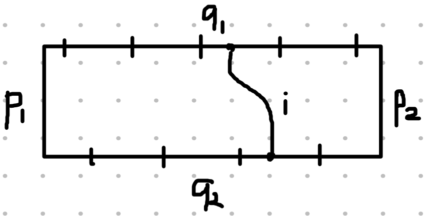
<u>Induction on |Δ(2)|</u>

Lemma 19.4: rd. dgrms. of rank i+1 are A-maps

Lemma 19.5: p is A-periodic sect. of rd. dgrm. of rank i+1.
A is simple of rank i+1, or
A is a period of rank $\leqslant$ i+1 and there are no A-comp. cells with p
=> p is smooth of rank |A|

Theorem 19.1: B(A, n) is infinite
+ other applications

Lemma 18.6



Δ – reduced circ. diagram of rank i

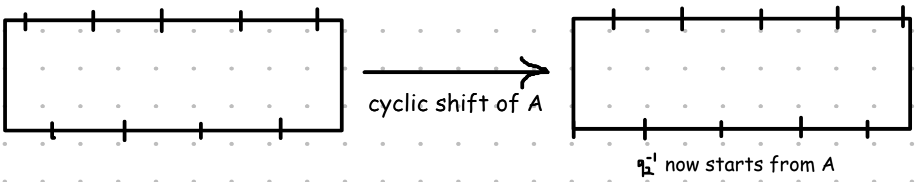
A - simple in rank i

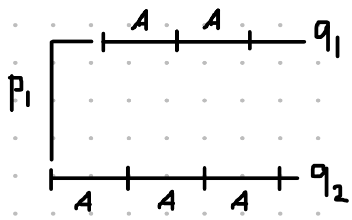
$\varphi(q_1),\ \varphi(q_2)^{-1}$ — A-periodic

$$|p_\nu| < \alpha|A|, \quad |q_\nu| > \left(\frac{5}{6}h+1\right)|A| \implies q_1 \text{ and } q_2 \text{ are A-compatible}$$

Proof

1)





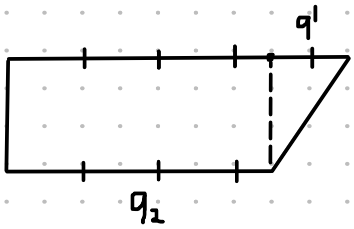
$\implies$

$$|p_1| < \left(\frac{1}{2}+\alpha\right)|A|, \quad |p_2| < \alpha|A|,$$

$|q_\nu| > \frac{5}{6}h\,|A|,\quad q_1,\ q_2^{-1}$ start with A,

$$\varphi(p_1) \overset{i}{\neq} 1$$

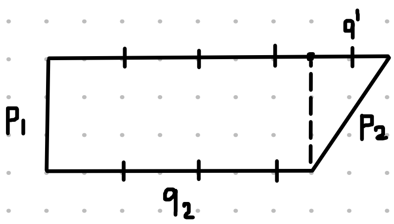
2) $|q_1| \geq |q_2|$ without loss of generality

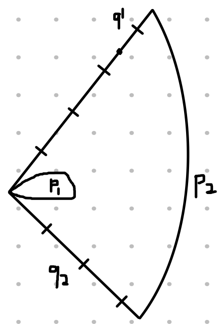


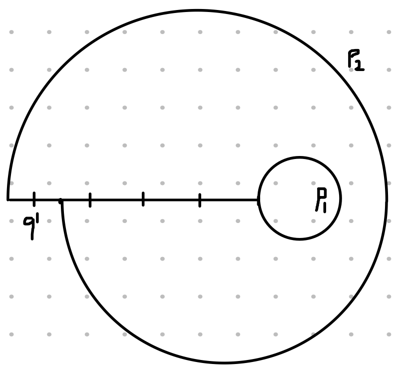
$$|q'| < \left(\frac{1}{\beta} - 1\right)|q_2| + |A|$$

(by using T17.1)

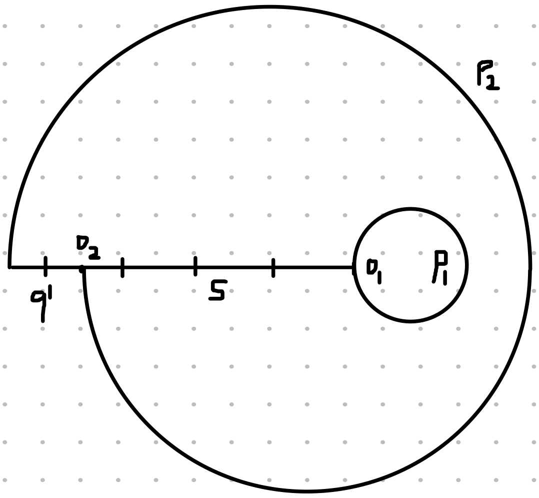
3)



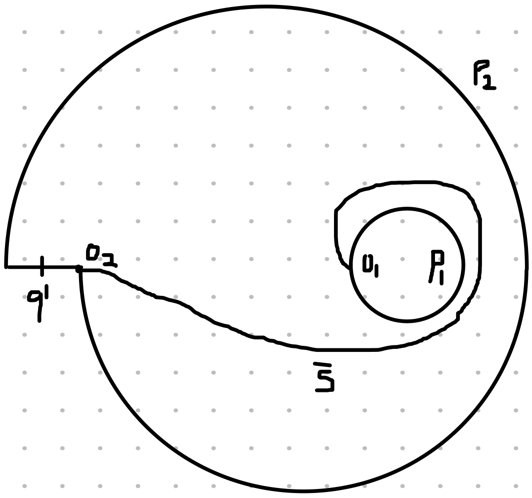




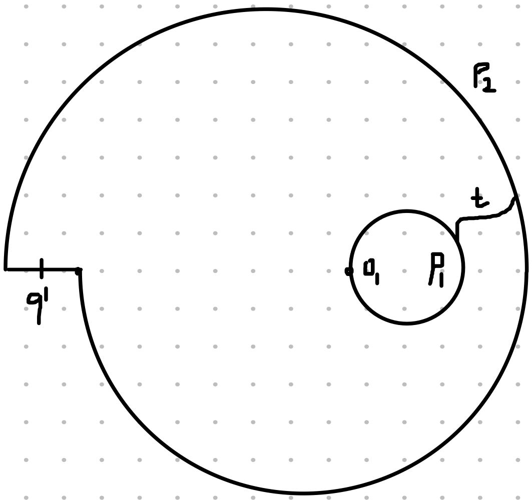
not reducing anything (yet)



$\xrightarrow{\text{after reducing}}$



$$\varphi(s) \overset{i}{=} \varphi(\bar{s})$$
$$|s| \geq |q_2|$$

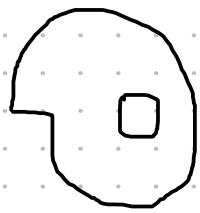


(other possible situations)

$$|s'| < |t| + \frac{1}{2}(|p_1| + |p_2|) < \gamma\left(\frac{1}{\beta} - 1\right)|q_2| + 2|A|$$

We can assume that s' ends in o_2

$$\varphi(\bar{s}) \overset{i}{=} \varphi(p_1)^f \varphi(s')$$



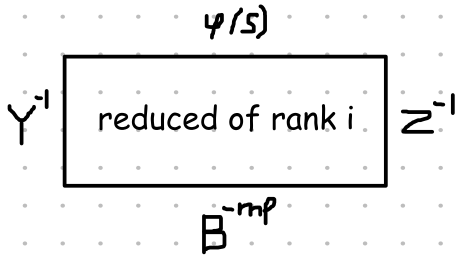
$\varphi(p_1) \overset{i}{=} Y B^m Y^{-1}$, B - simple in rank i or period of rank $k \leq i$

$|B^m| < \frac{1}{\beta} |\varphi(p_1)| < \frac{2}{3}|A|$ (L13.3 + L19.4, 19.5 => smooth, T17.1 => upper bound)

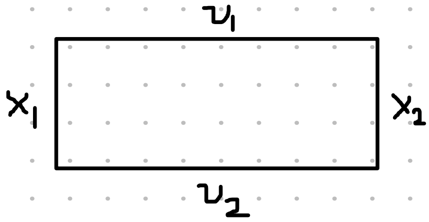
$|Y| < |A|$ (L18.5)

$$\Downarrow$$

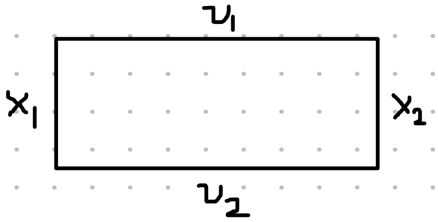
$$\varphi(s) \overset{i}{=} \varphi(\bar{s}) \overset{i}{=} Y B^{mf} \underbrace{Y^{-1} \varphi(s')}_{Z} \implies Y^{-1} \varphi(s) Z^{-1} B^{-mf} \overset{i}{=} 1$$



$\longrightarrow$



4)



$$|x_1| < |A|$$

$$|X_2| \leq |Y^{-1}| + |\varphi(S')| \leq \gamma\left(\frac{1}{\bar\beta} - 1\right)|q_2| + 3|A|$$

$$|v_1| \geq |S| \geq |q_2| > \frac{5}{6}h|A|$$

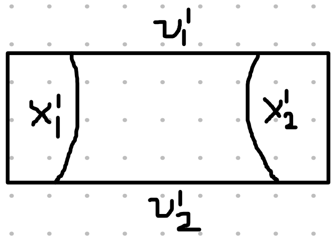
v_1 is A-periodic => smooth (A - simpl. in rk. i) $\quad$ v_2 is B-periodic => smooth (v_2 may change)

T17.1:

$$\bar\beta|v_1| \leq |x_1| + |v_2| + |X_2| \implies |v_2| > \bar\beta|q_2| - |x_1| - |X_2|$$

$$\implies |v_2| > |q_2|\left(\bar\beta - \gamma\left(\frac{1}{\bar\beta} - 1\right)\right) - 4|A|$$

rank(v_1) = |A|, rank(v_2) = |B|, |A| > |B|



Apply L17.5:

$$M = \frac{1}{\gamma}(|x_1| + |x_2| + |B|) < \left(\frac{1}{\bar\beta} - 1\right)|q_2| + \frac{5}{\gamma}|A|$$

$$\boxed{|q_2| > \frac{5}{6}h|A|}$$

$$|v_1| - M \geq |q_2| - M > 0$$

$$|v_2| - M > |q_2|\left(\bar\beta - (\gamma+1)\left(\frac{1}{\bar\beta} - 1\right)\right) - \left(4 + \frac{5}{\gamma}\right)|A| > 0$$

$$\implies$$

$$|X'_*| < \alpha|B|, \quad |v'_*| > |v_*| - M$$

$$|v_1'| > |v_1| - M > \frac{3}{4}h|A|, \quad |v_2'| > |v_2| - M > \frac{3}{4}h|A| > h|B|$$

5)

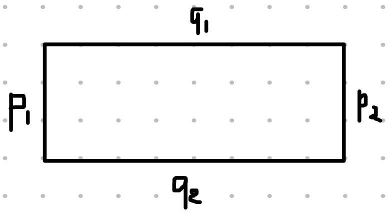
L17.4 $\implies$ j = rank(Γ') < |B| $\implies$ B is simple in rank j (period is simple in smaller ranks)

$|A| + |B| < |A| + |A|$ $\implies$ apply L18.8 $\implies$ $A \overset{i}{=} wB^{\pm1}w^{-1}$, $\quad |B| < |A|$

contr. with simplicity in rank i

Lemma 18.8

$\boxed{\text{Ind. on } |A| + |B|}$



Δ – reduced circ. diagram of rank i

A, B - simple in rank i, $\quad |A| \geq |B|$

$\varphi(q_1)$ – A-periodic, $\varphi(q_2)$ – B-periodic

$$|p_*| < \alpha|B|, \quad |q_1| > \frac{3}{4}h|A|, \quad |q_2| > h|B|$$

$$\implies$$

$$A \overset{i}{=} wB^{\pm1}w^{-1}$$