

Fuchsian groups

(course outline)

(November 8, 2021 to January 31, 2022)

Oleg Bogopolski

Düsseldorf university, Germany

This course is devoted to Fuchsian groups, which play an important role in different areas of mathematics: in the theory of Riemann surfaces, in number theory (modular forms), in dense sphere packings, in dynamical systems, and in geometric group theory.

The goal of this course is to give a detailed introduction to Fuchsian groups and their generalisations. The course is addressed to PhD students and researchers at the Institute of Mathematics, University of Szczecin.

Prerequisites: Basic knowledge in analysis and group theory.

Part I. Hyperbolic geometry

Lecture 1. The hyperbolic plane \mathbb{H} . The group of Möbius transformations of \mathbb{H} . Geodesic lines in \mathbb{H} .

Lecture 2. Some formulas for computing hyperbolic distances: cross-ratio formula, Log-and-Hyp formulas.

Lecture 3. The isometry group of \mathbb{H} . Hyperbolic area. Angles. Gauß-Bonnet formula. Hyperbolic trigonometry. Hyperbolic Pythagoras formula.

Part II. Fuchsian groups

Lecture 4. A classification of elements of $\mathrm{PSL}_2(\mathbb{R})$. Dynamic properties of elliptic, parabolic, and loxodromic elements.

Lecture 5. Three equivalent definitions of Fuchsian groups.

Lecture 6. Limit points of Fuchsian groups. Some algebraic properties of Fuchsian groups.

Lecture 7. Elementary Fuchsian groups.

Lecture 8. Jorgensen inequality. A discreteness criterium for subgroups of $\mathrm{PSL}_2(\mathbb{R})$.

Part III. Fundamental domains of Fuchsian groups

Lecture 9. Definition and some properties of a fundamental domain of a Fuchsian group.

Lecture 10. Dirichlet domains.

Lecture 11. Some theorems about the limit set $\Lambda(G)$ of a Fuchsian group G .

Lecture 10. Generators of Fuchsian groups arising from Dirichlet domains. Poincaré theorem.

Part IV. Arithmetic Fuchsian groups

Lecture 11. Quaternion algebras and quaternion Fuchsian groups.

Lecture 12. Arithmetic Fuchsian groups.

Part V. Hyperbolic groups

Lecture 13. Equivalent definitions and some properties of hyperbolic groups.

Lecture 14. Small cancellation groups as a source of hyperbolicity.

Lecture 15. Rips construction.

Tutorials: **15** hours.

The book [1] below gives a nice introduction to Fuchsian groups. The book [2] is more advanced and contains more information. The book [3] is a handbook for geometric group theorists; Chapter III of this book is devoted to hyperbolic groups. The book [4] is a handbook for combinatorial group theorists; Chapter V is devoted to small cancellation groups. The short paper [5] is important.

Literatur

- [1] Svetlana Katok, *Fuchsian groups*, The university of Chicago press, Chicago, 1992.
- [2] Alan F. Beardon, *The geometry of discrete groups*, Graduate Texts in Mathematics, No. **91**, Springer, 1983.
- [3] Martin Bridson, André Haefliger, *Metric spaces of non-positive curvature*, Springer, 1999.
- [4] Roger C. Lyndon, Pauln Schupp, *Combinatorial group theory*, Springer, Berlin, 1977.
- [5] Eliath Rips, *Subgroups of small cancellation groups*, Bull. London Math. Soc., **14** (1982), 45-47.