Global solvability of the Navier-Stokes equations in spaces based on sum-closed frequency sets

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Abstract

We prove existence of global regular solutions for the 3D Navier-Stokes equations with (or without) Coriolis force for a class of initial data u_0 in the space $\mathrm{FM}_{\sigma,\delta}$, i.e. for functions whose Fourier image $\widehat{u}_0$ is a vector-valued Radon measure and that are supported in sum-closed frequency sets with distance δ from the origin. In our main result we establish an upper bound for admissible initial data in terms of the Reynolds number, uniform on the Coriolis parameter Ω. In particular this means that this upper bound is linearly growing in δ. This implies that we obtain global in time regular solutions for large (in norm) initial data u_0 which may not decay at space infinity, provided that the distance δ of the sum-closed frequency set from the origin is sufficiently large.

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1 Introduction and main results

In this paper we consider the 3D Navier-Stokes equations with Coriolis force

$$\left\{\begin{array}{rcll} \partial_t u - \nu\Delta u + \Omega\mathbf{e}_3\times u + (u\cdot\nabla)u &=& -\nabla p & \text{in } \mathbb{R}_+\times\mathbb{R}^3,\\ \text{div } u &=& 0 & \text{in } \mathbb{R}_+\times\mathbb{R}^3,\\ u|_{t=0} &=& u_0 & \text{in } \mathbb{R}^3, \end{array}\right. \tag{1.1}$$

where $\mathbf{e}_3=(0,0,1)^T$, ν is the viscosity coefficient, and $\Omega\in\mathbb{R}$ is the Coriolis parameter, which is twice the angle velocity of the rotation.

Initial data u_0 is required to be an element of the space FM, that is the Fourier transform of M, which is the space of finite $\mathbb{C}^3$-valued Radon measures on $\mathbb{R}^3$. By the Riesz representation theorem it is known that M is the topological dual of

$$C_\infty(\mathbb{R}^3, \mathbb{C}^3) := \{f \in C(\mathbb{R}^3, \mathbb{C}^3) : f(x) \to 0 \text{ if } |x| \to \infty\}.$$

Note that FM equipped with the canonical norm $\|f\|_{\mathrm{FM}} := \|\mathcal{F}^{-1} f\|_{\mathrm{M}}$ is a Banach space, where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform. In particular we will consider initial values u_0 with Fourier image $\widehat{u}_0$ supported in sum-closed frequency sets, which are defined for general space dimensions $n = 1, 2, 3, \ldots$ as follows:

Definition 1.1. We say that $F \subseteq \mathbb{R}^n$ is a sum-closed frequency set in $\mathbb{R}^n$, if

(i) F is closed,

(ii) $0 \notin F$,

(iii) $F + F := \{x + y; x, y \in F\} \subseteq F \cup \{0\}$.

For a sum-closed frequency set with distance $\delta > 0$ from zero in the sequel we write F_δ. The class of all sum-closed frequency sets in $\mathbb{R}^n$ is denoted by $\mathscr{F}^n$.

Typical examples of sum-closed frequency sets are:

(i) Countable sum-closed frequency sets in $\mathbb{R}^n$ for which pairwise distances between frequency vectors are uniformly bounded from zero. This case corresponds to almost periodic initial data. The Cauchy problem for Navier-Stokes equations with almost periodic initial data was considered in [10] where local in time solvability was proven without restrictions on frequency sets.

(ii)

$$\mathbb{Z}^n \setminus \{0\},$$

or more general

$$F := \left\{ \sum_{j=1}^{n} m_j a_j;\ m = (m_1, \ldots, m_n) \in \mathbb{Z}^n \right\} \setminus \{0\},$$

where $a = \{a_1, \ldots, a_n\}$ represents a basis of $\mathbb{R}^n$. This case corresponds to periodic initial data. Indeed, $\operatorname{supp} \widehat{u}_0$ is contained in the above F for some a if and only if u_0 is periodic. Clearly, this is a special case

of (i).

(iii)

$$\{x \in \mathbb{R}^n : x_j \geq \delta\} \quad \text{for } j \in \{1, \dots, n\},\ \delta > 0.$$

Note that this example provides non real-valued initial data only.

Now let $F_\delta \in \mathscr{F}^3$. The main aim of this paper is to prove the existence of global regular solutions to (1.1) for initial data $u_0 \in \mathrm{FM}$ with $\operatorname{supp} \widehat{u}_0 \subseteq F_\delta$ and norm less than a number $M = M(\delta, \nu)$ which does not depend on the Coriolis parameter $\Omega \in \mathbb{R}$. Existence of solutions with norms uniformly bounded in Ω in spaces including functions nondecaying at infinity are essential in studies of statistical properties of turbulence, see e.g. [16, 19], and in the analysis of fast oscillating singular limits for system (1.1), see [1], [2] and [6].

As another interesting outcome of our approach, we obtain explicit dependence of M on the distance δ (and ν). In fact, we show that

$$M(\delta, \nu) = c_0 \nu \delta$$

with an explicitely given numerical constant c_0. This implies that we can prove the existence of global regular solutions to (1.1) for large initial data u_0 in FM, provided that $\operatorname{supp} \widehat{u}_0 \in F_\delta$ with δ is sufficiently large. We also emphasize that in our approach the case $\Omega = 0$ is not excluded, i.e. all the results presented here are valid for the standard Navier-Stokes equations without Coriolis force.

We note that in the case $\Omega = 0$ global existence of regular solutions for small initial data is proved in various function spaces. However, initial data are always assumed to decay at space infinity or to be periodic in space. For example, if the L^3-norm of initial data is small, then there is a global regular solution [14, 13, 7]. Although for the 2-dimensional problem it is known that there is a global regular solution for every bounded initial value [12, 17], it seems that there is no literature studying global solvability in the 3-dimensional case for nondecaying and nonperiodic initial data. In the situation of periodic boundary conditions, Chemin and Gallagher [4] constructed global regular solutions for system (1.1) in the case of $\Omega = 0$ for a certain class of initial data. Their approach relies on a splitting of the solution in a 2D part and a part satisfying a perturbed Navier-Stokes system. Admissible data need to satisfy a smallness condition of nonlinear type, which seems to be difficult to verify in general.

The case of 3D Navier-Stokes equations with large initial data characterized by uniformly large initial vorticity was considered in [1],

[2] and [6] in the L^2-setting in the case of periodic boundary conditions and cylindrical domains. It was shown that for sufficienly large Ω independent of the size of initial data in L^2, weak solutions of the 3D Navier-Stokes equations are in fact global in time strong solutions. The method of proving global regularity relies on the analysis of fast singular oscillating limits (singular limit $\Omega \to \infty$), nonlinear averaging methods and lemmas on restricted convolutions. This leads to the condition that Ω must be large in order to get global regular solutions for large initial data. There are no assumptions on 3D initial data besides that Ω is a fixed large parameter. Initial data can have arbitrary low and high frequency components.

In the present paper we prove global solvability of the 3D Navier-Stokes equations with initial data in spaces of functions nondecreasing at infinity based on sum-closed frequency sets provided that the distance from the origin of support of its Fourier image is sufficienly large. The latter condition can be interpreted as highly oscillating initial data. The Cauchy problem for Navier-Stokes equations ($\Omega = 0$) with highly oscillating initial data in Besov spaces is discussed in [3]. The property that highly oscillating initial data lead to global solutions to Navier-Stokes equations was implicitly contained in the papers of Kato and Fujita ([5], [15]). We note that our results on global regularity presented in this paper cover new spaces of functions for initial data nondecreasing at infinity which also contain almost periodic initial data. Our approach is based on a subtle splitting of the integral in the mild formulation of system (1.1), and the fact that the growth bound of the heat semigroup in FM tends to $-\infty$ if we increase the distance δ of the sum-closed frequency set to the origin. In order to get the independence of our results on Ω we use the fact that the Poincaré-Riesz semigroup associated to the Coriolis term $\Omega e_3 \times u$ is uniformly bounded in Ω in $\mathrm{FM}/\mathbb{C}^3$. This result is shown in [9]. There Matsui and the first three authors of this paper constructed a local-in-time classical solution to (1.1) uniformly in Ω.

We proceed with a rigorous statement of our main results. For $F_\delta \in \mathscr{F}^3$ we define the space

$$\mathrm{FM}_{\sigma,\delta} := \{f \in \mathrm{FM} : \operatorname{div} f = 0,\ \operatorname{supp} \widehat{f} \subseteq F_\delta\}. \tag{1.2}$$

Observe that here actually $\mathrm{FM} = \mathrm{FM}^3$, i.e. $f \in \mathrm{FM}^3$ is a $\mathbb{C}^3$-valued function and we enhance FM^3 with the norm

$$\|f\|_{\mathrm{FM}^3} := (\sum_{j=1}^{3} \|f_j\|_{\mathrm{FM}}^2)^{1/2}.$$

However, since it will always be clear from the context what we mean, in the sequel we will write FM, $\mathrm{FM}_{\sigma,\delta}$, etc. also for the vector-valued versions. It is easy to see that $\mathrm{FM}_{\sigma,\delta}$ is a closed subspace of FM. In Section 2 we shall give a more detailed discussion of the spaces FM and $\mathrm{FM}_{\sigma,\delta}$. In particular we will recall some of the results for the heat and the Poincaré-Riesz semigroup obtained in [9]. We also recall that $\mathrm{BC}(G,X)$ denotes the space of bounded continuous functions on $G \subseteq \mathbb{R}^n$ with values in a Banach space X.

Now we state main results of this paper.

Theorem 1.2. *Let $\nu, \delta > 0$, $\Omega \in \mathbb{R}$, $F_\delta \in \mathscr{F}^3$, and $u_0 \in \mathrm{FM}_{\sigma,\delta}$. Then, if*

$$\|u_0\|_{\mathrm{FM}} < \nu\delta/4K, \tag{1.3}$$

where

$$K = \sqrt{3}\left(\frac{\mathrm{e}^{-1}}{\sqrt{2}} + \frac{3\mathrm{e}^{3/2}}{2}\right) \approx 12.09433\,,\ ^{1} \tag{1.4}$$

there exists a unique global mild solution $u \in \mathrm{BC}([0,\infty), \mathrm{FM}_{\sigma,\delta})$ of the Navier-Stokes equations (1.1) satisfying

$$\|u(t) - u_0\|_{\mathrm{FM}} \to 0 \quad \textit{if} \quad t \to 0,$$

and

$$\|u(t)\|_{\mathrm{FM}} \le 2\mathrm{e}^{-\nu\delta^2 t}\|u_0\|_{\mathrm{FM}}, \quad t \ge 0.$$

Relation (1.3) implies that $\|u_0\|_{\mathrm{FM}}$ can be large, provided that the distance δ of F_δ from the origin is sufficiently large.

In the same way as in [8] we also obtain

Theorem 1.3. *Assume that the conditions of Theorem 1.2 hold and let u be the global mild solution obtained there. If we set*

$$p(t) = \Omega(-\Delta)^{-1/2}(R_2u^1(t) - R_1u^2(t)) + \sum_{j,k=1}^{3} R_jR_ku^j(t)u^k(t)$$

for the pressure p, where R_j denotes the Riesz operator associated to the symbol $i\xi_j/|\xi|$ for $j = 1,2,3$, then the pair $(u, \nabla p)$ is the unique classical solution to (1.1).

[1] This constant can be optimized to $K \le 2.5$; see Remark 3.2 (1).

Remark 1.4. (a) We note that the condition (1.3) can be written in terms of nondimensional physical parameter known as Reynolds number Re:

$$Re = \frac{\|u_0\|_{\mathrm{FM}}}{\nu\delta} < 1/4K. \tag{1.5}$$

The Reynolds number defined by (1.5) is based on velocity scale for initial data $\|u_0\|_{\mathrm{FM}}$, viscosity ν and a characteristic length scale $1/\delta$ ($1/\delta$ is a length scale since δ is a distance from the origin in frequency space). The above theorem establishes global solvability provided that initial Reynolds number satisfies the condition $Re < 1/4K$, where K is a nondimensional constant given by (1.4).

(b) Let $\|u_0\|_{\mathrm{FM}}$ be given. Then representation (1.4) allows an exact numerical determination of the distance δ, i.e. how far $\widehat{u}_0$ must be supported from the origin, in order to get global regular solutions.

(c) It is clear that in the case $\Omega = 0$ all the results remain true for arbitrary dimension $n \in \mathbb{N}$, modulo changing constants. This is even true for $\Omega \neq 0$, if we replace the rotation matrix J by a suitable skew-symmetric matrix.

(d) Observe that $\xi \in \operatorname{supp} \widehat{u}_0 \subseteq F$ implies that $-\xi \in \operatorname{supp} \widehat{u}_0 \subseteq F$, if the initial data u_0 is real-valued. Thus, the consideration of such initial data requires that $-F = F$ for the related sum-closed frequency set F. By the fact that F has a positive distance from 0 a simple argumentation shows that F must be a periodic lattice of the form given in example (ii) right after Definition 1.1.

Let us discuss two examples of (possibly "large") initial data covered by our results:

(1) Let $h \in L^1(\mathbb{R}^3)$ such that $\operatorname{supp} h \subseteq F_\delta := \{x \in \mathbb{R}^3 : x_3 \geq \delta\}$. Set $u_0 := P\mathcal{F}^{-1}h \in \mathrm{FM}_{\sigma,\delta}$, where P denotes the Helmholtz projection in $\mathbb{R}^3$. Then, by Lemma 2.2(i) we have

$$\|u_0\|_{\mathrm{FM}} \leq \|\mathcal{F}^{-1}h\|_{\mathrm{FM}} = \|h\|_1.$$

Thus, if

$$\delta > 4K\|h\|_1/\nu = 10\|h\|_1/\nu\,,\ ^2$$

there exists a global regular solution of (1.1) to u_0.

(2) Next, for any $d > 0$ we see that $F_d := d\mathbb{Z}^3 \setminus \{0\} \in \mathscr{F}^3$. We set

[2]Here we took the optimized K obtained in Remark 3.2 (1).

$u_0 = Pv$, where

$$v := \sum_{j=1}^{\infty} a_j \mathrm{e}^{i\lambda_j \cdot x}, \quad x \in \mathbb{R}^3,\ \lambda_j \in F_d,$$

and $(a_j)_{j\in\mathbb{N}}$ can be any sequence in $\mathbb{R}^3$ such that $\sum_{j=1}^{\infty} |a_j| < \infty$. Then,

$$\widehat{v} = (2\pi)^{3/2} \sum_{j=1}^{\infty} a_j \delta(\xi - \lambda_j) \quad \text{and} \quad \|u_0\|_{\mathrm{FM}} \le (2\pi)^{3/2} \sum_{j=1}^{\infty} |a_j|,$$

where here δ denotes the Dirac delta distribution. Thus $u_0 \in \mathrm{FM}_{\sigma,d}$. Furthermore, we see that we can get global regular solutions of (1.1) for the initial values of the form $u_0 = Pv$, if the vectors of the frequencies $\lambda_j = (\lambda_{j,1}, \lambda_{j,2}, \lambda_{j,3})$ satisfy

$$\sqrt{\lambda_{j,1}^2 + \lambda_{j,2}^2 + \lambda_{j,3}^2} \ \ge\ 4K(2\pi)^{3/2} \sum_{j=k}^{\infty} |a_k|/\nu, \quad j = 1, 2, \dots .$$

We organized this paper as follows. In Section 2 we list basic properties of the spaces FM and $\mathrm{FM}_{\sigma,\delta}$ and for the linear operators associated to (1.1). In particular we recall from [9] estimates for the heat semigroup, the Poincaré-Riesz semigroup, and the Helmholtz projection in the spaces FM. By utilizing these estimates, in Section 3 then we prove our main results Theorem 1.2 and Theorem 1.3.

2 Preliminaries

Here we recall some of the basic results on the space FM and the linear operators associated to (1.1) obtained in [9]. As an easy consequence of these results at the end of this section we will state the cruicial exponential decay rate estimate for the heat semigroup on the space $\mathrm{FM}_{\sigma,\delta}$.

By the Riesz representation theorem it is well known that each linear form L on $C_\infty(\mathbb{R}^n, \mathbb{C})$ can be represented as

$$L(f) = \int_{\mathbb{R}^n} f\nu \mathrm{d}\eta,$$

where $\nu : \mathbb{R}^n \to \mathbb{C}$ is an η-measurable function satisfying $|\nu(x)| = 1$, $x \in \mathbb{R}^n$, and η is a finite positive Radon measure on $\mathbb{R}^n$. Recall that

η is a finite Radon measure if it is a positive Borel-regular measure satisfying $\eta(\mathbb{R}^n) < \infty$ (see [18]). Therefore, the element μ of the space M associated to L is given by

$$\mu(O) := \int_O \nu \mathrm{d}\eta, \quad O \subseteq \mathbb{R}^n \text{ open},$$

and the norm on M, that is

$$\|\mu\|_{\mathrm{M}} := \sup\left\{ |\int_{\mathbb{R}^n} f\nu \mathrm{d}\eta|;\, f \in C_c(\mathbb{R}^n),\, \|f\|_\infty \leq 1 \right\},$$

is called the total variation norm, where $C_c(O)$ denotes the space of continuous functions with compact support in O. Now, for $\mu \in \mathrm{M}$ let η_μ be the associated finite positive Radon measure and let ν_μ be the associated η_μ-measurable function satisfying $|\nu_\mu(x)| = 1$. For a η_μ-a.e. bounded and Borel measurable function ψ we may define

$$\mu\lfloor\psi(O) := \int_O \psi\nu_\mu \mathrm{d}\eta_\mu, \quad O \subseteq \mathbb{R}^n \text{ open}.$$

Since this can be written as

$$\mu\lfloor\psi(O) = \int_O (\chi_{\{\psi>0\}} - \chi_{\mathbb{R}^n\setminus\{\psi>0\}})\nu_\mu|\psi|\mathrm{d}\eta_\mu,$$

it is clear that $\mu\lfloor\psi \in \mathrm{M}$. If $B \subseteq \mathbb{R}^n$ is a Borel set and $\psi = \chi_B$ is the characteristic function then we simply write $\mu\lfloor B$ for $\mu\lfloor\chi_B$. The total variation measure $|\mu|$ of $\mu \in \mathrm{M}$ is defined by

$$|\mu|(O) := \sup\left\{ |\int_{\mathbb{R}^n} f\nu \mathrm{d}\eta|;\, f \in C_c(O),\, \|f\|_\infty \leq 1 \right\},$$

for open $O \subseteq \mathbb{R}^n$. It follows easily from the definition that we have the relations $\mu = \eta_\mu\lfloor\nu_\mu$, $|\mu| = \eta_\mu$, $|\mu\lfloor\psi| \leq |\mu|\lfloor|\psi|$, and $(\mu\lfloor\psi)\lfloor\phi = \mu\lfloor(\psi\phi)$, if ϕ is another $|\mu|$-a.e. bounded Borel measurable function.

In order to define multipliers with symbols not necessarily continuous at 0 we also introduce the space

$$\mathrm{M}_0 := \{\mu \in \mathrm{M};\, \mu\lfloor\{0\} = 0\},$$

i.e. $\mu \in \mathrm{M}_0$ has no point mass at the origin. Since $\mu\lfloor\{0\} = 0$ is equivalent to say that $\lim_{r\downarrow 0} |\mu|(B_r(0)) = 0$, where $B_r(0)$ denotes the open ball with radius r and center 0, it is easy to see that M_0 is a closed subspace of M.

The space

$$\mathrm{M}^n := \{\mu = (\mu_1, \dots, \mu_n) : \ \mu_j \in \mathrm{M}, \ j = 1, \dots, n\}$$

with norm

$$\|\mu\|' := \sup\{|\langle \mu, f\rangle|; \ f \in (C_c(\mathbb{R}^n))^n, \ \|f\|_\infty \le 1\}, \tag{2.1}$$

where

$$\langle \mu, f\rangle := \sum_{j=1}^n \mu \lfloor f_j$$

we call the space of finite vector-valued Radon measures. For our purposes it will be more suitable to enhance M with the norm

$$\|\mu\|_{\mathrm{M}^n} := \left(\sum_{j=1}^n \|\mu_j\|_{\mathrm{M}}^2\right)^{1/2}.$$

If we fix the vector-valued $\|\cdot\|_\infty$ norm in (2.1) as

$$\|f\|_\infty := \left(\sum_{j=1}^n \|f_j\|_\infty^2\right)^{1/2}, \quad f \in (L^\infty(\mathbb{R}^n))^n,$$

then the relation between the two norms on M is

$$\|\cdot\|' \le \|\cdot\|_{\mathrm{M}^n} \le \sqrt{n}\|\cdot\|'.$$

For simplicity in the sequel we will write M for M^n, $\|\cdot\|_{\mathrm{M}}$ for $\|\cdot\|_{\mathrm{M}^n}$, $C_c(\mathbb{R}^n)$ for $(C_c(\mathbb{R}^n))^n$, etc.

The Fourier transform on the Schwartz space $\mathcal{S}(\mathbb{R}^n)$ we denote by

$$\widehat{f}(\xi) = \mathcal{F}f(\xi) := \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} \mathrm{e}^{-ix\cdot\xi} f(x)\mathrm{d}x, \quad f \in \mathcal{S}(\mathbb{R}^n),$$

and its extension to the space of tempered distributions $\mathcal{S}'(\mathbb{R}^n)$ is defined in the usual way. Since M as the dual of $C_\infty(\mathbb{R}^n)$ can be regarded as a subspace of $\mathcal{S}'(\mathbb{R}^n)$ we may define the space

$$\mathrm{FM} := \{\widehat{\mu} : \ \mu \in \mathrm{M}\}.$$

Equipped with the norm

$$\|f\|_{\mathrm{FM}} := \|\widehat{f}\|_{\mathrm{M}} = \|\mathcal{F}^{-1}f\|_{\mathrm{M}},$$

FM is a Banach space and we have that FM $\subseteq$ BUC($\mathbb{R}^n$), where BUC($\mathbb{R}^n$) denotes the space of bounded uniformly continuous functions on $\mathbb{R}^n$. An important closed subspace of FM is

$$\mathrm{FM}_0 := \{\widehat{f};\, f \in \mathrm{M}_0\}.$$

Note that this space is isomorphic to the quotient space FM/$\mathbb{C}^n$. In fact FM = $\mathrm{FM}_0 \oplus \mathbb{C}^n$. Furthermore, in [9] it is proved that FM_0 is even a subspace of the homogeneous Besov space $\dot{\mathcal{B}}^0_{\infty,1}(\mathbb{R}^n)$. For $\sigma = (\sigma_{jk}) \in [C(\mathbb{R}^n \setminus \{0\}) \cap L^\infty(\mathbb{R}^n)]^{n\times n}$ we define

$$(Op(\sigma)f)_j := \mathcal{F}^{-1}\sum_{k=1}^{n} \widehat{f_k}\lfloor\sigma_{jk}, \quad j = 1,\dots,n,\ f \in \mathrm{FM}_0. \tag{2.2}$$

As an immediate consequence of the definition of FM and FM_0 we obtain the following multiplier result, which is essential for the uniformness of our main results in the Coriolis parameter Ω.

Lemma 2.1. [9, Lemma 2.2] *Let $\sigma = (\sigma_{jk}) \in [C(\mathbb{R}^n \setminus \{0\}) \cap L^\infty(\mathbb{R}^n)]^{n\times n}$. Then $Op(\sigma)$ as defined in (2.2) is bounded on* FM_0 *and we have*

$$\|Op(\sigma)f\|_{\mathrm{FM}} \le \|\,|\sigma|\,\|_\infty \|f\|_{\mathrm{FM}}, \quad f \in \mathrm{FM}_0,$$

where $|M| := \sup_{y\in\mathbb{R}^n} |My|$ for a matrix $M \in \mathbb{R}^{n\times n}$ and $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^n$. Furthermore, if σ is also continuous at the origin, then the operator $Op(\sigma)$ is a bounded operator on FM and the above estimate holds for all $f \in FM$.

From now on we restrict our considerations to 3 space dimensions. Note that in the context as introduced above the Helmholtz projection is given by

$$(Pf)_j = \mathcal{F}^{-1}\sum_{k=1}^{3} \widehat{f_j}\lfloor\sigma(P)_{jk}, \quad j = 1,2,3,\ f \in \mathrm{FM}_0,$$

where $\sigma(P)_{jk} = (\delta_{jk} - \frac{\xi_j\xi_k}{|\xi|^2})$, the heat semigroup as

$$\mathrm{e}^{t\Delta}f = \mathcal{F}^{-1}(\widehat{f}\lfloor\mathrm{e}^{-t|\cdot|^2}), \quad f \in \mathrm{FM}^1,$$

and the Poincaré-Riesz operator by

$$Sf := PJPf, \quad f \in \mathrm{FM}_0,$$

where

$$J = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Observe that $\sigma(P)$ is orthogonal, $\sigma(S)$ is skew-symmetric, and thus $\sigma(\mathrm{e}^{tS})$ is unitary on $\mathbb{C}^3$. As a consequence of Lemma 2.1 we therefore obtain the following estimates (see [9, Lemma 2.5, Lemma 2.9]).

Lemma 2.2. (i) *The operators P, S, and e^{tS} are bounded on FM_0. In particular we have*

$$\|Pf\|_{\mathrm{FM}} \le \|f\|_{\mathrm{FM}}, \quad f \in \mathrm{FM}_0,$$

$$\|\mathrm{e}^{tS} f\|_{\mathrm{FM}} \le \|f\|_{\mathrm{FM}}, \quad t \in \mathbb{R},\ f \in \mathrm{FM}_0.$$

(ii) *The family $\{e^{t\Delta}\}_{t\ge 0}$ is a bounded holomorphic C_0-semigroup of contractions on FM and FM_0. Moreover, it satisfies*

$$\|\partial_j e^{t\Delta} f\|_{\mathrm{FM}} \le (2te)^{-1/2} \|f\|_{\mathrm{FM}}, \quad f \in \mathrm{FM},\ t > 0,\ j = 1,2,3.$$

Next, for $F_\delta \in \mathscr{F}^3$ let $\mathrm{FM}_{\sigma,\delta}$ be defined as in (1.2). It is clear that this is a closed subspace of FM_0 and that it is invariant under the operations P, S, and $\mathrm{e}^{t\Delta}$. The main results in this paper rely essentially on the following two facts. Firstly,

$$\operatorname{supp} \widehat{f}, \operatorname{supp} \widehat{g} \subseteq F_\delta \quad \Longrightarrow \quad \operatorname{supp} \widehat{fg} \subseteq F_\delta \cup \{0\}, \tag{2.3}$$

which is a consequence of the definition of sum-closed frequency sets (in particular of property (iii)). This property will be important for applying the contraction mapping principle in the next section. Essentially it implies that the space $\mathrm{FM}_{\sigma,\delta}$ is also invariant under the nonlinear operation arising from the convective term in system (1.1). Secondly, it relies on the following exponential decay rate estimate for the heat semigroup depending on the distance δ of F_δ from the origin.

Lemma 2.3. *Let $F_\delta \in \mathscr{F}^3$. The family $\{e^{t\Delta}\}_{t\ge 0}$ is a bounded holomorphic C_0-semigroup of contractions on $\mathrm{FM}_{\sigma,\delta}$ satisfying*

$$\|e^{t\Delta} f\|_{\mathrm{FM}} \le \mathrm{e}^{-t\delta^2} \|f\|_{\mathrm{FM}}, \quad f \in \mathrm{FM}_{\sigma,\delta},\ t > 0.$$

Proof. By virtue of $\operatorname{supp} \widehat{f} \subseteq F_\delta$ we obtain

$$\begin{aligned} \|\mathrm{e}^{t\Delta} f\|_{\mathrm{FM}} &= \|\widehat{f}\lfloor \mathrm{e}^{-t|\cdot|^2}\|_{\mathrm{M}} = \left|\widehat{f}\lfloor \mathrm{e}^{-t|\cdot|^2}\right|(F_\delta) \\ &\leq |\widehat{f}|\lfloor \mathrm{e}^{-t|\cdot|^2}|(F_\delta) = \int_{F_\delta} \mathrm{e}^{-t|\cdot|^2}\mathrm{d}|\widehat{f}| \\ &\leq \mathrm{e}^{-t\delta^2}\|f\|_{\mathrm{FM}}, \quad f \in \mathrm{FM}_{\sigma,\delta}. \end{aligned}$$

□

Remark 2.4. Of course the solenoidality is not essential for the assertion, i.e. Lemma 2.3 remains true if we replace $\mathrm{FM}_{\sigma,\delta}$ by $\mathrm{FM}_\delta := \{f \in \mathrm{FM} : \operatorname{supp} \widehat{f} \subseteq F_\delta\}$.

3 Global solutions

We proceed with the construction of global mild solutions to problem (1.1). To this end we define for $T > 0$,

$$\begin{aligned} G &: \mathrm{BC}([0,T), \mathrm{FM}^{3\times 3}) \to \mathrm{BC}([0,T), \mathrm{FM}_0), \\ Gf(t) &:= \int_0^t P \operatorname{div} \mathrm{e}^{(t-s)(\nu\Delta - \Omega S)} f(s)\mathrm{d}s, \end{aligned}$$

where $\mathrm{FM}^{3\times 3}$ is equipped with the norm

$$\|f\|_{\mathrm{FM}} := \left(\sum_{j,k=1}^{3} \|f_{jk}\|_{\mathrm{FM}}^2\right)^{1/2}, \quad f \in \mathrm{FM}^{3\times 3}.$$

Observe that G is well-defined, since by the presence of the operator div we may always replace f by $f - \mathcal{F}^{-1}(\widehat{f}\lfloor\{0\}) \in \mathrm{FM}_0$. The crucial step is to prove

Proposition 3.1. *Let $F_\delta \in \mathscr{F}^3$ and let $f \in \mathrm{BC}([0,\infty), \mathrm{FM}^{3\times 3})$ such that $\operatorname{supp} \widehat{f} \subseteq F_\delta \cup \{0\}$. Then*

$$\sup_{t>0} \|\mathrm{e}^{\nu\delta^2 t} Gf(t)\|_{\mathrm{FM}} \leq \frac{K}{\nu\delta} \sup_{t>0} \|\mathrm{e}^{2\nu\delta^2 t} f(t)\|_{\mathrm{FM}}, \tag{3.1}$$

with K given in (1.4). Moreover, we have

$$\|Gf(t)\|_{\mathrm{FM}} \to 0 \quad \textit{if} \quad t \to 0. \tag{3.2}$$

Proof. One key for the proof lies in a splitting of the integral in the term $\mathrm{e}^{\nu\delta^2 t}Gf(t)$ into an integration over the two subintervals $[0, 3t/4]$ and $[3t/4, t]$. We denote the two resulting (sub-) integrals by $I_1(t)$ and $I_2(t)$, respectively.

First we estimate I_2. For this part observe that by Lemma 2.2(i) and (ii) we obtain

$$\begin{aligned}
\|I_2(t)\|_{\mathrm{FM}} &\leq \mathrm{e}^{\nu\delta^2 t}\int_{3t/4}^{t}\|P\mathrm{e}^{-\Omega(t-s)S}\mathrm{div}\ \mathrm{e}^{(t-s)\nu\Delta}f(s)\|_{\mathrm{FM}}\mathrm{d}s \\
&\leq \sqrt{3}\frac{\mathrm{e}^{\nu\delta^2 t}}{\sqrt{2\nu\mathrm{e}}}\int_{3t/4}^{t}\frac{1}{\sqrt{t-s}}\mathrm{e}^{-2\nu\delta^2 s}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}}\mathrm{d}s \\
&\leq \sqrt{3}\frac{\mathrm{e}^{\nu\delta^2 t}}{\sqrt{2\nu\mathrm{e}}}\mathrm{e}^{-\nu\delta^2 t/2}\int_{3t/4}^{t}\frac{1}{\sqrt{t-s}}\mathrm{d}s\sup_{t>0}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}} \\
&\leq \sqrt{3}\frac{1}{\sqrt{2\nu\mathrm{e}}}\sqrt{t}\mathrm{e}^{-\nu\delta^2 t/2}\sup_{s>0}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}} \qquad (3.3) \\
&\leq \sqrt{3}\frac{1}{\sqrt{2\nu\mathrm{e}}}\sup_{t>0}\left(\sqrt{t}\mathrm{e}^{-\nu\delta^2 t/2}\right)\sup_{s>0}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}} \\
&= \frac{\sqrt{3}\,\mathrm{e}^{-1}}{\sqrt{2}\,\nu\delta}\sup_{s>0}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}}
\end{aligned}$$

Next consider I_1. Here we pick $r > 0$, to be fixed later, and assume first that $t \geq 4r$. We will see that the introduction and the right choice of r will be another key for the proof. Again an application of Lemma 2.2(i) and (ii) yields

$$\begin{aligned}
\|I_1(t)\|_{\mathrm{FM}} &\leq \mathrm{e}^{\nu\delta^2 t}\int_{0}^{3t/4}\|\mathrm{div}\,\mathrm{e}^{r\nu\Delta}\mathrm{e}^{(t-s-r)\nu\Delta}f(s)\|_{\mathrm{FM}}\mathrm{d}s \\
&\leq \sqrt{3}\frac{\mathrm{e}^{\nu\delta^2 t}}{\sqrt{2r\nu\mathrm{e}}}\int_{0}^{3t/4}\|\mathrm{e}^{(t-s-r)\nu\Delta}f(s)\|_{\mathrm{FM}}\mathrm{d}s.
\end{aligned}$$

Note that always $t - s - r \geq 0$ for $s \in [0, 3t/4]$ when $t \geq 4r$. Thus, employing Lemma 2.3 this calculation continues to the result

$$\begin{aligned}
\|I_1(t)\|_{\mathrm{FM}} &\leq \sqrt{3}\frac{\mathrm{e}^{\nu\delta^2 t}}{\sqrt{2r\nu\mathrm{e}}}\int_{0}^{3t/4}\mathrm{e}^{-(t-s-r)\nu\delta^2}\mathrm{e}^{-2\nu\delta^2}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}}\mathrm{d}s \\
&\leq \sqrt{3}\frac{\mathrm{e}^{\nu\delta^2 r}}{\sqrt{2r\nu\mathrm{e}}}\int_{0}^{3t/4}\mathrm{e}^{-\nu\delta^2 s}\mathrm{d}s\sup_{s>0}\|\mathrm{e}^{2\nu\delta^2 s}f(s)\|_{\mathrm{FM}}.
\end{aligned}$$

In order to avoid growth in δ the last line shows how we have to choose

r, namely e.g. as $r = 1/2\nu\delta^2$. This implies

$$\begin{aligned} \|I_1(t)\|_{\mathrm{FM}} &\leq \sqrt{3}\delta \frac{1-\mathrm{e}^{-3\nu\delta^2 t/4}}{\nu\delta^2} \sup_{s>0} \|\mathrm{e}^{2\nu\delta^2 s} f(s)\|_{\mathrm{FM}} \qquad (3.4) \\ &\leq \frac{\sqrt{3}}{\nu\delta} \sup_{s>0} \|\mathrm{e}^{2\nu\delta^2 s} f(s)\|_{\mathrm{FM}}, \quad t \geq 2/\nu\delta^2. \end{aligned}$$

The case where $t \leq 2/\nu\delta^2$ is easily estimated as

$$\begin{aligned} \|I_1(t)\|_{\mathrm{FM}} &\leq \mathrm{e}^{\nu\delta^2 t} \int_0^{3t/4} \|\mathrm{div}\ \mathrm{e}^{(t-s)\nu\Delta} f(s)\|_{\mathrm{FM}} \mathrm{d}s \\ &\leq \sqrt{3} \frac{\mathrm{e}^{\nu\delta^2 t}}{\sqrt{2\nu\mathrm{e}}} \int_0^{3t/4} \frac{1}{\sqrt{t-s}} \mathrm{e}^{-2\nu\delta^2 s} \|\mathrm{e}^{2\nu\delta^2 s} f(s)\|_{\mathrm{FM}} \mathrm{d}s \\ &\leq \sqrt{3} \frac{2\mathrm{e}^{\nu\delta^2 t}}{\sqrt{2\nu\mathrm{e}}} \frac{3\sqrt{t}}{4} \sup_{s>0} \|\mathrm{e}^{2\nu\delta^2 s} f(s)\|_{\mathrm{FM}} \qquad (3.5) \\ &\leq \frac{3\sqrt{3}\,\mathrm{e}^{3/2}}{2\nu\delta} \sup_{s>0} \|\mathrm{e}^{2\nu\delta^2 s} f(s)\|_{\mathrm{FM}}. \end{aligned}$$

Observe that $\sqrt{3} < 3\sqrt{3}\,\mathrm{e}^{3/2}/2$ implies that $\sup_{t>0} \|I_1(t)\|_{\mathrm{FM}} = \sup_{t\leq 1/2\nu\delta^2} \|I_1(t)\|_{\mathrm{FM}}$. Hence, taking supremum on the left hand sides, i.e. on $\|\mathrm{e}^{\nu\delta^2 t} Gf(t)\|_{\mathrm{FM}} \leq \|I_1(t)\|_{\mathrm{FM}} + \|I_2(t)\|_{\mathrm{FM}}$ over $t > 0$ we arrive at (3.1). Relation (3.2) is an easy consequence of the estimates (3.3), (3.4), and (3.5). Hence the proposition is proved. □

Finally we turn to the proof of Theorem 1.2. Let $F_\delta \in \mathscr{F}^3$, $u_0 \in \mathrm{FM}_{\sigma,\delta}$, and set

$$B_{u_0,\delta} := \{u \in \mathrm{BC}([0,\infty), \mathrm{FM}_{\sigma,\delta}) : \|u\|_\delta \leq 2\|u_0\|_{\mathrm{FM}}\},$$

where $\|u\|_\delta := \sup_{t>0} \|\mathrm{e}^{\nu\delta^2 t} u(t)\|_{\mathrm{FM}}$. By applying the Helmholtz projection we rewrite system (1.1) in the operatorial form

$$\begin{cases} \partial_t u - \nu\Delta u + \Omega S u + P(u\cdot\nabla)u &= -\nabla p \quad \text{in } (0,\infty), \\ u|_{t=0} &= u_0. \end{cases} \qquad (3.6)$$

Observe that the existence of a (mild) solution to (3.6) is equivalent to the existence of a fixed point for the nonlinear map H defined by

$$\begin{aligned} Hu(t) &:= \mathrm{e}^{t(\nu\Delta - \Omega S)} u_0 - \int_0^t P\,\mathrm{div}\,\mathrm{e}^{(t-s)(\nu\Delta-\Omega S)} u(s)u(s)^T \mathrm{d}s \\ &= \mathrm{e}^{t\nu\Delta} u_0 - [G(uu^T)](t), \quad u \in B_{u_0,\delta}. \end{aligned}$$

Note that $\operatorname{supp} uu^T \subseteq F_\delta \cup \{0\}$ by virtue of (2.3). Thus, Lemma 2.2(i), Lemma 2.3, and Proposition 3.1 imply

$$\begin{aligned}\|Hu\|_\delta &\leq \|u_0\|_{\mathrm{FM}} + \frac{K}{\nu\delta}\sup_{s>0}\|\mathrm{e}^{2\nu\delta^2 t}u(s)u(s)^T\|_{\mathrm{FM}} \\ &\leq \|u_0\|_{\mathrm{FM}} + \frac{K}{\nu\delta}\|u\|_\delta^2 \\ &\leq \|u_0\|_{\mathrm{FM}} + \frac{4K}{\nu\delta}\|u_0\|_{\mathrm{FM}}^2, \quad u \in B_{u_0,\delta},\end{aligned}$$

and we observe that $H(B_{u_0,\delta}) \subseteq B_{u_0,\delta}$ whenever the relation $\|u_0\|_{\mathrm{FM}} \leq \nu\delta/4K$ is satisfied. To see that H is a contraction we again employ estimate (3.1), which yields that

$$\begin{aligned}&\|Hu - Hv\|_\delta \\ &\leq \frac{K}{\nu\delta}\sup_{t>0}\|\mathrm{e}^{2\nu\delta^2 t}(u(s)u(s)^T - v(s)v(s)^T)\|_{\mathrm{FM}} \\ &\leq \frac{K}{\nu\delta}\sup_{t>0}\|\mathrm{e}^{2\nu\delta^2 t}(u(s)(u(s)-v(s))^T + (u(s)-v(s))v(s)^T)\|_{\mathrm{FM}} \\ &\leq \frac{K}{\nu\delta}(\|u(s)\|_\delta + \|v(s)\|_\delta)\|(u(s)-v(s))\|_\delta \\ &\leq \frac{K}{\nu\delta}\|u_0\|_{\mathrm{FM}}\|(u(s)-v(s))\|_\delta.\end{aligned}$$

Hence, H is a contraction in $B_{u_0,\delta}$ if

$$\|u_0\|_{\mathrm{FM}} < \nu\delta/4K \tag{3.7}$$

is satisfied. Therefore, if $F_\delta \in \mathscr{F}^n$ and $u_0 \in \mathrm{FM}_{\sigma,\delta}$, such that relation (3.7) holds, the contraction mapping principle yields the existence of a unique fixed point $u \in B_{u_0,\delta}$ of H. As an obvious consequence of (3.2) and Lemma 2.2(ii) we also have that $u(t) \to u_0$ in FM. Thus the proof of Theorem 1.2 is now completed.

We conclude this note with two remarks and an announcement.

Remark 3.2. (1) The constant K in (1.4) is not optimal. For instance, if we split the integral in the proof of Proposition 3.1 at α instead at $3/4$ and set $r = \beta/2\nu\delta^2$ instead of $r = 1/2\nu\delta^2$, we can obtain

$$K = K(\alpha,\beta) = \sqrt{\frac{3}{2}}\left(\mathrm{e}^{-1/2}\sqrt{\frac{1-\alpha}{2\alpha-1}} + \max\left\{\sqrt{\beta}\,\mathrm{e}^{2\beta}\frac{\alpha}{\sqrt{1-\alpha}}, \frac{\mathrm{e}^{\beta/2}}{\sqrt{\beta}}\right\}\right)$$

for $(\alpha, \beta) \in (1/2, 1) \times (0, \infty)$. Minimizing this expression with respect to (α, β) yields

$$K \leq 2.5\,.$$

(2) Similar results to Theorem 1.2 and Theorem 1.3 can be obtained, if we replace $\mathrm{FM}_{\sigma,\delta}$ by the corresponding subspace of the homogeneous Besov space $\dot{\mathcal{B}}^0_{\infty,1}(\mathbb{R}^3)$, i.e. the space

$$\{f \in \dot{\mathcal{B}}^0_{\infty,1}(\mathbb{R}^3) : \operatorname{div} f = 0,\ \operatorname{supp} \widehat{f} \subseteq F_\delta\}$$

for $F_\delta \in \mathscr{F}^3$. Indeed, all the assertions remain true with the only exception that the constant K in (1.4) then depends on Ω, and it is expected that $K(\Omega) \to \infty$ if $\Omega \to \infty$.

(3) An inspection of the proof of Theorem 1.2 shows that the crucial ingredient in our approach is the increasing exponential decay rate of the heat semigroup. This gives rise to the conjecture that this approach might work in much greater generality. In fact, it can be shown that the pair $(\mathrm{FM}_{\sigma,\delta}, -\Delta)$ can be replaced by many other pairs (X_δ, A_δ) such that A_δ is a Stokes operator on the space X_δ satisfying similar properties as $-\Delta$ on $\mathrm{FM}_{\sigma,\delta}$, in particular an increasing exponential decay rate for $\delta \to \infty$. Another concrete example for X_δ might be the scale $L^p_\sigma(D_\delta)$, where D_δ denotes a layer with thickness δ. A detailed discussion of this issue will be part of the content of the forthcoming paper [11]. There we will even demonstrate how our approach extends to more general semilinear equations.

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