STOKES AND NAVIER-STOKES EQUATIONS WITH PERFECT SLIP ON WEDGE TYPE DOMAINS

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Abstract. Well-posedness of the Stokes and Navier-Stokes equations subject to perfect slip boundary conditions on wedge type domains is studied. Applying the operator sum method we derive an $\mathcal{H}^\infty$-calculus for the Stokes operator in weighted L^p_γ spaces (Kondrat'ev spaces) which yields maximal regularity for the linear Stokes system. This in turn implies mild well-posedness for the Navier-Stokes equations, locally-in-time for arbitrary and globally-in-time for small data in L^p.

Contents

1. Introduction

We consider the Navier-Stokes equations subject to perfect slip boundary conditions given as

$$\left\{\begin{array}{rcll} u_t - \Delta u + \nabla p + (u\cdot\nabla)u &=& 0 & \text{in} \quad (0,T)\times G, \\ \operatorname{div u} &=& 0 & \text{in} \quad (0,T)\times G, \\ \nu\times\operatorname{curl} u = 0, \quad u\cdot\nu &=& 0 & \text{on} \quad (0,T)\times\partial G, \\ u(0) &=& u_0 & \text{in} \quad G. \end{array}\right. \tag{1.1}$$

Here

$$G = S_{\varphi_0}\times\mathbb{R}, \quad S_{\varphi_0} := \{(x_1,x_2)\in\mathbb{R}^2 :\ 0<x_1<\infty,\ 0<x_2<x_1\tan\varphi_0\}$$

represents a domain of wedge type and ν denotes the outer normal vector at ∂G. Fluid flow in wedge type domains is closely related to contact line problems arising in wetting and de-wetting phenomena. The idea is to locally transform a three-phase (liquid/gas/solid) contact line to a wedge domain by employing a suitable Hanzawa transformation, see e.g. [4] or section 4 in

[21]. Due to the free boundary of the fluid/gas interface this, however, leads usually to intricate quasilinear problems with dynamic boundary conditions in wedge domains. An analytical treatment of these problems appears very hard. In fact, it seems only the 'trivial' values of contact angles, that is $\varphi_0 = 0, \pi/2, \pi$, could be handled so far, cf. [26], [7], [25].

A major objective of this note is to show that at least for an 'easy' set of boundary conditions the fundamental equations of fluid dynamics are well-posed on a three-dimensional wedge for arbitrary angles $\varphi_0 \in (0, \pi)$. The strategy we pursue is as follows. In a first step we consider the parabolic resolvent problem

$$\left\{ \begin{array}{rcll} (\lambda - \Delta)u & = & f & \text{in} \quad G, \\ \nu \times \operatorname{curl} u & = & 0 & \text{on} \quad \partial G, \\ u \cdot \nu & = & 0 & \text{on} \quad \partial G. \end{array} \right. \tag{1.2}$$

in the Kondrat'ev space

$$L^p_\gamma(G, \mathbb{R}^3) := L^p(G, |x|^\gamma d(x_1, x_2, y), \mathbb{R}^3) \tag{1.3}$$

for appropriate $\gamma \in \mathbb{R}$. (Actually in a certain subspace of $L^p_\gamma(G)$, see Section 3.)

A common approach, which is also utilized here, is to transform this system (note that still $u = (u_1, u_2, u_3)$) to a layer by introducing polar coordinates and applying the Euler transformation. The resulting transformed system (see (2.4)-(2.7)) then can be handled by abstract results on operator sums, cf. [2], [5]. In our situation we apply suitable Kalton-Weis type theorems, cf. [11]. In fact, the corresponding transformed linear operator consists of a sum in which every summand admits a bounded $\mathcal{H}^\infty$-calculus. A specific feature here is that some of the operators are non-commuting (in the resolvent sense). Here we apply [22, Theorem 3.1] which represents a Kalton-Weis type theorem for the non-commuting case based on the Labbas-Terreni commutator condition, which was introduced in [13]. Hence the $\mathcal{H}^\infty$-calculus transfers to the full sum. This, in turn, yields this property to be valid for the Laplacian related to (1.2) as well.

Prüss and Simonett already successfully applied this method in [22] to the scalar version of problem (1.2) if Dirichlet boundary conditions on ∂G are imposed. In fact, Prüss and Simonett precisely recovered the results on maximal regularity for the Dirichlet-Laplacian on wedge type domains obtain before in [18] by Nazarov via direct methods based on the Green's function. The outcome in [18] also covers the case of Neumann boundary conditions.

Having the $\mathcal{H}^\infty$-calculus for the Laplacian corresponding to system (1.2) at hand we turn to the Stokes equations

$$\left\{\begin{array}{rcccc} u_t - \Delta u + \nabla p & = & f & \text{in} & (0,T)\times G, \\ \operatorname{div} \mathrm{u} & = & 0 & \text{in} & (0,T)\times G, \\ \nu \times \operatorname{curl} u = 0, \quad u\cdot\nu & = & 0 & \text{on} & (0,T)\times \partial G, \\ u(0) & = & 0 & \text{in} & G. \end{array}\right. \tag{1.4}$$

This is the point where the partial slip conditions become essential. In fact, for this type of boundary conditions it can be proved that the Helmholtz projection and the Laplacian commute. Thus the Stokes operator can be regarded as the part of the Laplacian in the solenoidal subspace $L^p_{\sigma,\gamma}(G)$, see Section 3. This immediately yields the $\mathcal{H}^\infty$-calculus also to hold for the Stokes operator corresponding to system (1.4). Our main result on the linearized system therefore reads as follows.

Theorem 1.1. *Assume that* $1<p<\infty$, $\gamma\in\mathbb{R}$, *and* $\varphi_0\in(0,\pi)$ *satisfy*

$$\min\left\{1, \left(\frac{\pi}{\varphi_0}-1\right)^2\right\} > \left(2-\frac{2+\gamma}{p}\right)^2 \tag{1.5}$$

Then the Stokes operator

$$\begin{aligned} &\mathcal{A}_S u := -\Delta u, \\ u\in D(\mathcal{A}_S) := \Big\{ &u\in L^p_{\sigma,\gamma}(G):\ \nu\times\operatorname{curl} u = 0,\ \nu\cdot u = 0 \text{ on } \partial G, \\ &u/|(x_1,x_2)|^2,\ \partial^\alpha u \in L^p_\gamma(G,\mathbb{R}^3)\ (\alpha\in\mathbb{N}_0^3,\ |\alpha|\le 2)\Big\} \end{aligned}$$

associated to system (1.4) admits a bounded $\mathcal{H}^\infty$*-calculus on* $L^p_{\sigma,\gamma}(G)$ *with* $\mathcal{H}^\infty$*-angle* $\phi^\infty_{\mathcal{A}_S} < \pi/2$.

In the case $\gamma = 0$ by duality and interpolation the Stokes operator is defined for arbitrary $1<p<\infty$ and $\varphi_0\in(0,\pi)$. This immediately yields

Corollary 1.2. *Assume that* $\gamma=0$ *and let* $1<p<\infty$ *and* $\varphi_0\in(0,\pi)$ *be arbitrary. Then the Stokes operator* $\mathcal{A}_S$ *admits a bounded* $\mathcal{H}^\infty$*-calculus on* $L^p_\sigma(G)$ *with* $\mathcal{H}^\infty$*-angle* $\phi^\infty_{\mathcal{A}_S}<\pi/2$.

Theorem 1.1 in particular implies that $\mathcal{A}_S$ generates a bounded analytic C_0-semigroup on $L^p_{\sigma,\gamma}(G)$ and that it has maximal regularity. Hence we also have

Corollary 1.3. *Suppose the assumptions of Theorem 1.1 hold and let* $J = (0,T)$ *with* $T\in(0,\infty)$. *Then for each* $f\in L^p(J,L^p_{\sigma,\gamma}(G))$ *there exists a unique solution* $u\in L^p(J,L^p_{\sigma,\gamma}(G))$ *of (1.4) possessing the regularity*

$$u, u/|\cdot|^2, \partial_t u, \nabla^2 u \in L^p(J, L^p_\gamma(G,\mathbb{R}^3)).$$

In particular, the map $[u\mapsto f]$ *defines an isomorphism between the corresponding spaces.*

Note that the fact that Helmholtz projection and Laplacian commute in the perfect slip setting has already been proved and utilized by Mitrea and Monniaux in [15] and [16]. Indeed, in [16] well-posedness for system (1.1) is studied in the context of bounded (graph) Lipschitz domains. For the linear (Hodge-) Stokes operator it is proved that it is the generator of an analytic C_0-semigroup in L^p provided p is within the usual range between $((3+\varepsilon)', 3+\varepsilon)$, cf. [15]. Although it is the same set of equations, we think that the outcomes of [15], [16] and the underlying note are in some sense not comparable. The roughness of the boundary forces the authors in [15], [16] to work in Hodge spaces (i.e. $\operatorname{curl} u, \operatorname{div} u \in L^p$ instead $\nabla u \in L^p$) which in that case do not coincide with corresponding Sobolev spaces. The results obtained here, however, provide full Sobolev regularity as well as the full range $p \in (1,\infty)$ (if $\gamma = 0$).

In combination with a general result from [9], Theorem 1.1 yields the following main result concerning the nonlinear system (1.1).

Theorem 1.4. (i) *(Existence and Uniqueness). Suppose* $u_0 \in L^r_\sigma(G)$, $r \geq 3$. *Then there is* $T_0 > 0$ *and a unique mild solution of (1.1) on* $[0, T_0)$ *such that*

$$u \in BC([0,T_0), L^r_\sigma) \cap L^q(0, T_0, L^p_\sigma)$$
$$t^{\frac{1}{q}} u \in BC([0,T_0), L^p_\sigma), \quad t^{\frac{1}{q}} u(t) \to 0 \quad (t \to 0)$$

with $\frac{2}{q} + \frac{3}{p} = \frac{3}{r}$, $q, p > r$. *There is a positive constant* ε *such that if* $\|u_0\|_3 < \varepsilon$ *then* $T_0 = \infty$.

(ii) *(Estimate for the blow-up). Let* $(0, T_*)$ *be the maximal interval such that* u *solves (1.1) in* $C((0,T_*), L^r_\sigma)$, $r > 3$. *Then*

$$\|u(s)\|_r \geq \frac{c}{(T_* - s)^{(r-3)/2r}}$$

with constant $c > 0$ *independent of* T_* *and* s.

Remark 1.5. We remark that by obvious modifications of the proofs our main results remain valid in case that the underlying domain is a two-dimensional wedge. Then we have $G = S_{\varphi_0} \subset \mathbb{R}^2$ and the boundary conditions take the form

$$\operatorname{curl} u = 0, \quad u \cdot \nu = 0,$$

where $\operatorname{curl} u = \partial_1 u_2 - \partial_2 u_1$ for a two dimensional vector field u.

We continue as follows. In Section 2 we transform (1.2) via polar coordinates and Euler transformation to a degenerate problem on a layer. In Section 3 we prove an $\mathcal{H}^\infty$-calculus for the related linear operator of the transformed system. In Section 4 it is demonstrated how this result transfers to the Stokes operator associated to (1.4), i.e., we prove Theorem 1.1. Finally, in Section 5 we show well-posedness of system (1.1), i.e. we prove Theorem 1.4.

2. Transformation of the parabolic linear problem

We consider a three-dimensional wedge as it is given in the introduction. In the first step we introduce cylinder coordinates, while in a second step we apply the Euler transformation. In a third step we rescale the appearing terms such that in the transformed setting we can work in unweighted L^p-spaces.

Let again $\varphi_0 \in (0, \pi)$ denote the angle of the wedge and set $I := (0, \varphi_0)$. The inverse of the transformation to polar coordinates we write as

$$\psi_P : \mathbb{R}_+ \times I \times \mathbb{R} \to G, \quad (r, \varphi, y) \mapsto (r \cos \varphi, r \sin \varphi, y) = (x_1, x_2, y).$$

Since we deal with vector fields, we also employ the standard orthogonal basis for cylinder coordinates in $\mathbb{R}^3$ given by

$$e_r = \begin{pmatrix} \cos \varphi \\ \sin \varphi \\ 0 \end{pmatrix}, \qquad e_\varphi = \begin{pmatrix} -\sin \varphi \\ \cos \varphi \\ 0 \end{pmatrix}, \qquad e_y = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The orthogonal transformation matrix $\mathcal{O}$ for the components of a vector field then reads

$$\mathcal{O} = \begin{pmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In radial direction we apply the Euler transformation $r = e^x$, where in slight abuse of notation $x \in \mathbb{R}$ denotes the new variable. We set $\Omega := \mathbb{R} \times I \times \mathbb{R}$ and

$$\psi_E : \Omega \to \mathbb{R}_+ \times I \times \mathbb{R}, \quad (x, \varphi, y) \mapsto (e^x, \varphi, y) =: (r, \varphi, y).$$

It is then clear that

$$\psi := \psi_P \circ \psi_E : \Omega \to G$$

is a diffeomorphism.

Having introduced all required transformations we define the pull back of the solution u of (1.2) as

$$v := \Theta^* u := e^{-\beta x} \mathcal{O}^{-1} u \circ \psi.$$

Thus the corresponding push forward reads

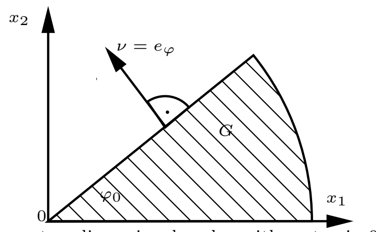
$$u = \Theta_* v = \mathcal{O}(e^{\beta x} v) \circ \psi^{-1}. \tag{2.1}$$

Next, we compute the transformed differential operators. We obtain

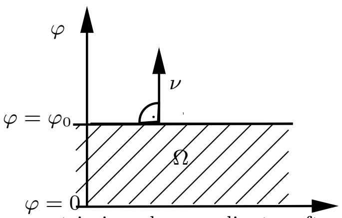
$$\Theta^*(\Delta u) = e^{(\beta-2)x} \begin{pmatrix} e^{2x}\partial_y^2 v_x - P(\partial_x) v_x + \partial_\varphi^2 v_x - v_x - 2\partial_\varphi v_\varphi \\ e^{2x}\partial_y^2 v_\varphi - P(\partial_x) v_\varphi + \partial_\varphi^2 v_\varphi - v_\varphi + 2\partial_\varphi v_x \\ e^{2x}\partial_y^2 v_y - P(\partial_x) v_y + \partial_\varphi^2 v_y \end{pmatrix},$$

with the polynomial

$$P(\partial_x) := -(\partial_x^2 + (2\beta)\partial_x + \beta^2).$$



two-dimensional wedge with vertex in 0 and angle φ_0



strip in polar coordinates after applying polar coordinate and Euler transformation

In order to absorb the factor $e^{(\beta-2)x}$ we also set

$$g = (g_x, g_\varphi, g_y) := \tilde{\Theta}^* f := e^{2x}\Theta^* f. \tag{2.2}$$

Note that then

$$\int_{\mathbb{R}} |g(x,\varphi,y)|^p dx = \int_0^\infty |r^{2-\beta}\mathcal{O}^{-1} f(\psi_P(r,\varphi,y))|^p \frac{dr}{r}.$$

Thus, as it is already utilized in [22], the choice $p(2-\beta) = \gamma + 2$, that is $\beta = 2 - (\gamma+2)/p$, allows for a treatment of the transformed system in unweighted spaces. We also have

$$\begin{aligned}
(\operatorname{div} u)\circ\psi &= (\operatorname{div}\Theta_* v)\circ\psi \\
&= e^{(\beta-1)x}(\beta v_x + v_x + \partial_x v_x + \partial_\varphi v_\varphi) + e^{\beta x}\partial_y v_y, \qquad (2.3)\\
\tilde{\Theta}^*(\operatorname{curl} u) &= e^{2x}\left(e^{-x}\partial_\varphi v_y - \partial_y v_\varphi\right) e_r + e^{2x}\left(\partial_y v_x - e^{-x}(\beta v_y + \partial_x v_y)\right) e_\varphi \\
&\quad + e^x\left((\beta+1)v_\varphi + \partial_x v_\varphi - \partial_\varphi v_x\right) e_y.
\end{aligned}$$

It remains to transform the boundary conditions

$$u\cdot\nu = 0, \qquad \nu\times\operatorname{curl} u = 0 \quad \text{on } \partial G.$$

They are equivalent to

$$u\cdot\nu = 0, \quad (\operatorname{curl} u)\cdot\tau_1 = 0, \quad (\operatorname{curl} u)\cdot\tau_2 = 0 \quad \text{on } \partial G$$

for two linearly independent tangential vectors τ_1, τ_2. It is nearby to choose

$$\tau_1 = e_r, \quad \tau_2 = e_y$$

at $\varphi = 0$ and $\varphi = \varphi_0$ respectively. This yields

$$\partial_\varphi v_x = 0, \quad v_\varphi = 0, \quad \partial_\varphi v_y = 0 \quad \text{on } \partial\Omega = \mathbb{R}\times\{0,\varphi_0\}\times\mathbb{R}.$$

Altogether the transformed system reads as

$$e^{2x}\lambda v_x - e^{2x}\partial_y^2 v_x + P(\partial_x)v_x - \partial_\varphi^2 v_x + v_x + 2\partial_\varphi v_\varphi = g_x \quad \text{in } \Omega, \tag{2.4}$$

$$e^{2x}\lambda v_\varphi - e^{2x}\partial_y^2 v_\varphi + P(\partial_x)v_\varphi - \partial_\varphi^2 v_\varphi + v_\varphi - 2\partial_\varphi v_x = g_\varphi \quad \text{in } \Omega, \tag{2.5}$$

$$e^{2x}\lambda v_y - e^{2x}\partial_y^2 v_y + P(\partial_x)v_y - \partial_\varphi^2 v_y = g_y \quad \text{in } \Omega, \tag{2.6}$$

$$\partial_\varphi v_x = 0, \;\; v_\varphi = 0, \;\; \partial_\varphi v_y = 0 \quad \text{on } \partial\Omega. \tag{2.7}$$

In the next section we prove strong well-posedness for this system. As in [22] one difficulty here is to handle the non-standard differential operator $e^{2x}(\lambda - \partial_y^2)$ and the fact that this operator and $P(\partial_x)$ do not commute.

3. $\mathcal{H}^\infty$-calculus and maximal L^p-regularity

The aim of this section is to prove an $\mathcal{H}^\infty$-calculus for the linear operator corresponding to problem (1.2). This will be derived by building up the full operator by its single parts via the operator sum method. In fact, we will prove that each single part admits a bounded $\mathcal{H}^\infty$-calculus. Based on commutative [11] and non-commutative [22] Kalton-Weis theorems this property transfers to the full linear operator.

First let us fix the notation used throughout this note. Let X be a Banach space. For a domain $\Omega \subset \mathbb{R}^n$ let $C_c^\infty(\Omega, X)$ denote the space of smooth and compactly supported X-valued functions defined on Ω and $C_{c,\sigma}^\infty(\Omega, \mathbb{R}^n) := \{\varphi \in C_c^\infty(\Omega, \mathbb{R}^n) : \operatorname{div} \varphi = 0\}$. For $1 \le p \le \infty$ and a measure space (S, Σ, μ), we write $L^p(S, \mu, X)$ for the usual Bochner-Lebesgue space. If $\Omega \subset \mathbb{R}^n$ and μ is the (Borel-) Lebesgue measure, we also write $L^p(\Omega, X)$. The symbol $W^{k,p}(\Omega, X)$ denotes the Sobolev space of order $k \in \mathbb{N}_0$, where $W^{0,p} := L^p$. Now, for $\gamma \in \mathbb{R}$ set

$$\mu_\gamma(U) := \int_U |(x_1, x_2)|^\gamma \, d(x_1, x_2, y) \quad (U \in \mathscr{B}(\mathbb{R}^3)),$$

where $\mathscr{B}(\mathbb{R}^3)$ denotes the Borel σ-algebra. On the wedge $G = S_{\varphi_0} \times \mathbb{R}$ we define weigthed Bochner-Lebesgue and Sobolev spaces via

$$\begin{aligned} L^p_\gamma(G, X) &:= L^p(G, \mu_\gamma, X), \\ W^{k,p}_\gamma(G, X) &:= \{u \in L^p_\gamma(G, X) |\ \partial^\alpha u \in L^p_\gamma(G, X)\ (\alpha \in \mathbb{N}_0^3,\ |\alpha| \le k)\} \end{aligned}$$

for $k \in \mathbb{N}$, $1 \le p \le \infty$.

Given a Banach spaces X, Y the space of bounded linear operators from X to Y shall be denoted by $\mathscr{L}(X, Y)$, where $\mathscr{L}(X) := \mathscr{L}(X, X)$. The subclass of isomorphisms is denoted by $\mathscr{L}_{is}(X, Y)$ or $\mathscr{L}_{is}(X)$, respectively. If A is a linear operator in X then $D(A)$, $R(A)$ and $N(A)$ stand for its domain, range and kernel respectively, where $\sigma(A)$, $\sigma_p(A)$, $\sigma_c(A)$, $\sigma_r(A)$, $\rho(A)$ mean its spectrum, point spectrum, continuous spectrum, residual spectrum and its resolvent set. We denote a complex sector of angle $\phi \in (0, \pi)$ by

$$\Sigma_\phi := \{z \in \mathbb{C} :\ z \neq 0,\ |\arg(z)| < \phi\}.$$

Definition 3.1. A closed linear operator A in a Banach space X is called *sectorial*, if

(i) $\overline{D(A)} = X$, $N(A) = \{0\}$, $\overline{R(A)} = X$,
(ii) $(-\infty, 0) \subset \rho(A)$ and there is a $c > 0$ such that $\|t(t + A)^{-1}\|_{\mathscr{L}(X)} \le c$ for all $t > 0$.

In this case it is well-known (Taylor expansion), that there exists a $\phi \in [0,\pi)$ such that the uniform estimate in (ii) extends to all $\lambda \in \Sigma_{\pi-\phi}$. We call

$$\phi_A := \inf\{\phi : \rho(-A) \supset \Sigma_{\pi-\phi}, \sup_{\lambda\in\Sigma_{\pi-\phi}} \|\lambda(\lambda+A)^{-1}\|_{\mathscr{L}(X)} < \infty\}$$

the *spectral angle* of A. The class of sectorial operators is denoted by $\mathcal{S}(X)$.

Next we introduce the notion of a bounded $\mathcal{H}^\infty$-calculus. For a comprehensive introduction to this concept we refer to [3], [11], and [12].

For $\sigma \in (0,\pi)$ we define

$$\mathcal{H}^\infty(\Sigma_\sigma) := \{f : \Sigma_\sigma \to \mathbb{C} : f \text{ holomorphic}, \|f\|_\infty < \infty\}$$

where

$$\|f\|_\infty := \sup\{|f(z)| : z \in \Sigma_\sigma\}.$$

For $\rho(z) := z/(1+z)^2$ we define the subalgebra

$$\begin{aligned}\mathcal{H}_0(\Sigma_\sigma) :=\{& f \in \mathcal{H}^\infty(\Sigma_\sigma) : \exists C,\varepsilon > 0\,\forall z \in \Sigma_\sigma : \\ & |f(z)| \le C|\rho(z)|^\varepsilon\}.\end{aligned}$$

Let A be a sectorial operator in X with spectral angle ϕ_A. Let $\sigma \in (\phi_A,\pi)$ and $\psi \in (\phi_A,\sigma)$. The path $\Gamma := (\infty,0]e^{i\psi} \cup [0,\infty)e^{-i\psi}$ stays with the only possible exception at zero in the resolvent set of A. Hence

$$f(A) := \frac{1}{2\pi i}\int_\Gamma f(\mu)(\mu - A)^{-1}d\mu \tag{3.1}$$

is a well-defined element in $\mathscr{L}(X)$ for every $f \in \mathcal{H}_0(\Sigma_\sigma)$. The above formula defines an algebra homomorphism

$$\Phi_A : \mathcal{H}_0(\Sigma_\sigma) \to \mathscr{L}(X), \quad f \mapsto f(A), \tag{3.2}$$

a so-called Dunford calculus. For general $f \in \mathcal{H}^\infty(\Sigma_\sigma)$ we set

$$f(A) := \rho(A)^{-1}(\rho f)(A),$$

which gives rise to a closed, densely defined operator in X.

Definition 3.2. The operator $A \in \mathcal{S}(X)$ is said to admit a *bounded $\mathcal{H}^\infty$-calculus on X*, if there exists $\sigma > \phi_A$ such that Φ_A given in (3.2) is bounded (w.r.t. the topologies on $\mathcal{H}^\infty(\Sigma_\sigma)$ and $\mathscr{L}(X)$). We denote by $\mathcal{H}^\infty(X)$ the class of operators admitting a bounded $\mathcal{H}^\infty$-calculus on X. The number ϕ_A^∞ denotes the infimum over all $\sigma > \phi_A$ such that Φ_A remains bounded and is called *$\mathcal{H}^\infty$-angle of A*.

Remark 3.3. It is well-known that, if $A \in \mathcal{H}^\infty(X)$, then Φ_A extends to a bounded algebra homomorphism from $\mathcal{H}^\infty(\Sigma_\sigma)$ to $\mathscr{L}(X)$ for $\sigma > \phi_A^\infty$, cf. [12].

In the abstract results applied below (see e.g. Proposition 3.6 and Proposition 3.9) the notions of class $\mathcal{HT}$ and of property (α) for Banach spaces appear. For its rigorous definition and relations to known properties we refer again to [3], [11], and [12]. Here we only remark that reflexive L^p spaces and their closed subspaces, hence all crucial spaces used in this note, enjoy these properties.

For $1 < p < \infty$ and $\Omega = \mathbb{R}^2 \times I$ we set

$$X := L^p\left(\Omega, \mathbb{R}^3\right).$$

Note that for the sake of convenience from now on we write the space variables in the order $(x, y, \varphi) \in \mathbb{R}^2 \times I$, but we keep the order of components as $v = (v_x, v_\varphi, v_y)$. Occasionally we also write $\mathbb{R}_y$, $\mathbb{R}_x$, I_φ to indicate the relation between domain and the corresponding variable.

We denote the norm on X by $\|\cdot\|$. Our full operator consists of the following single parts:

(1) We define B in $L^p(\mathbb{R})$ by means of

$$Bu(x) = P(\partial_x)u(x), \quad x \in \mathbb{R}, \quad u \in D(B) = W^{2,p}(\mathbb{R}).$$

Its spectrum is given by the parabola $P(i\mathbb{R})$, which is symmetric about the real axis, open to the right, and has its vertex in $a_0 := -\beta^2 \in \mathbb{R}$. It is known that $\omega + B \in \mathcal{H}^\infty(L^p(\mathbb{R}))$ for $\omega > -a_0$, with $\phi^\infty_{\omega+B} < \pi/2$, see [22]. The same is true for the canonical extension to X which we again denote by B. Note that $B - a_0$ is accretive in X, cf. [22].

(2) We denote by L_y the Laplacian in $L^p(\mathbb{R})$ in the y-variable:

$$L_y u(y) = -\Delta_y u(y) = -\partial_y^2 u(y),\ y \in \mathbb{R},\ u \in D(L_y) = W^{2,p}(\mathbb{R}).$$

The operator L_y admits an $\mathcal{H}^\infty$-calculus in $L^p(\mathbb{R})$ with $\phi^\infty_{L_y} = 0$. The spectrum is $\sigma(L_y) = [0, \infty)$. The same holds true for the canonical extension to X which we again denote by L_y. Furthermore L_y is accretive.

(3) We also have to deal with the multiplication operator M in $L^p(\mathbb{R})$ defined by

$$\begin{aligned} Mu(x) &= e^{2x}u(x), \quad x \in \mathbb{R}, \\ D(M) &= \{u \in L^p(\mathbb{R}) : (x \mapsto e^{2x}u(x)) \in L^p(\mathbb{R})\}. \end{aligned}$$

It is easy to see that also this operator admits a bounded $\mathcal{H}^\infty$-calculus with $\phi^\infty_M = 0$ and that we have $\sigma(M) = [0, \infty)$. Likewise the canonical extension of M to X enjoys the same properties and will again be denoted by M.

(4) We define $L_{N,D}$ in $L^p(I, \mathbb{R}^2)$ and L_N in $L^p(I)$ by

$$L_{N,D}v' := \begin{pmatrix} 1 - \partial_\varphi^2 & 2\partial_\varphi \\ -2\partial_\varphi & 1 - \partial_\varphi^2 \end{pmatrix} v', \ \ L_N v_y := -\partial_\varphi^2 v_y$$

on

$$\begin{aligned} &D(L_{N,D}) := \{v' = (v_x, v_\varphi) \in W^{2,p}(I, \mathbb{R}^2) :\ \partial_\varphi v_x = 0,\ v_\varphi = 0 \text{ on } \partial I\}, \\ &D(L_N) := \{v_y \in W^{2,p}(I) :\ \partial_\varphi v_y = 0 \text{ on } \partial I\}, \end{aligned}$$

respectively. So, $L_{N,D}$ is subject to the Neumann conditions of v_x and the Dirichlet conditions of v_φ and L_N to the Neumann conditions of v_y in (2.7). Furthermore, we set

$$Lv := \begin{pmatrix} L_{N,D} & 0 \\ 0 & L_N \end{pmatrix} v, \quad v \in D(L),$$
$$D(L) := \{v \in W^{2,p}(I, \mathbb{R}^3) : \partial_\varphi v_x = 0,\ v_\varphi = 0,\ \partial_\varphi v_y = 0 \text{ on } \partial I\}$$

in $L^p(I, \mathbb{R}^3)$. The spectrum of these operators can be determined explicitly. In fact, it is straight forward to verify that

$$\sigma_p(L_{N,D}) = \left\{ \left(\frac{\pi k}{\varphi_0} \pm 1 \right)^2 :\ k \in \mathbb{N} \right\} \cup \{1\}$$

with corresponding eigenfunctions

$$v_x^k(\varphi) := \cos\left(\frac{\pi k}{\varphi_0}\varphi\right), \quad v_\varphi^k(\varphi) := \pm \sin\left(\frac{\pi k}{\varphi_0}\varphi\right), \quad k \in \mathbb{N}_0,\ \varphi \in I.$$

Next, by well-known results on eigenvalues of the Neumann-Laplacian we obtain

$$\sigma(L_N) = \sigma_p(L_N) = \{0\} \cup \left\{ \frac{\pi^2}{\varphi_0^2} k^2 :\ k \in \mathbb{N} \right\}.$$

Consequently, $\sigma(L) = \sigma_p(L) = \sigma_p(L_{N,D}) \cup \sigma_p(L_N)$, that is

$$\sigma(L) = \{0\} \cup \{1\} \cup \left\{ \left(\frac{\pi k}{\varphi_0} \pm 1 \right)^2 :\ k \in \mathbb{N} \right\} \cup \left\{ \frac{\pi^2}{\varphi_0^2} k^2 :\ k \in \mathbb{N} \right\}. \tag{3.3}$$

Furthermore, it is not difficult to show that L admits a bounded $\mathcal{H}^\infty$-calculus on $L^p(I, \mathbb{R}^3)$ with $\phi_L^\infty = 0$. (This follows for instance by the spectral decomposition (Fourier series) and the fact that the collection of eigenfunctions of L represents a basis of $L^p(I, \mathbb{R}^3)$ for every $1 < p < \infty$.) Its canonical extension to X enjoys the same properties and will again be denoted by L.

As it is shown later on (see Lemma 4.1), the eigenvalue 0 will play no further rôle when dealing with the Stokes equations. Thus, we may exclude it which improves the spectral properties of L. Note that this is even essential for the applicability of Proposition 3.9 below. To exclude the corresponding eigenspace we set

$$L_0^p(I) := \left\{ u \in L^p(I) :\ \int_I u(\varphi) d\varphi = 0 \right\}.$$

The projection onto this subspace is given as

$$\pi_0 : L^p(I) \longrightarrow L_0^p(I), u \mapsto u - \frac{1}{|I|} \int_I u(\varphi) d\varphi.$$

Then

$$\Pi_0 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \pi_0 \end{pmatrix} \tag{3.4}$$

is the projection onto the new ground space

$$X_0 := L^p(\mathbb{R}^2, L^p(I) \times L^p(I) \times L_0^p(I)). \tag{3.5}$$

We obviously have $(1 - \Pi_0)X = L^p(\mathbb{R}^2, E_0)$ with

$$E_0 = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle, \tag{3.6}$$

hence the decomposition

$$X = X_0 \oplus L^p(\mathbb{R}^2, E_0). \tag{3.7}$$

This in particular implies $\Pi_0 L = L\Pi_0$. Thus $L_0 := L|_{X_0}$ is well-defined from $D(L_0) := \Pi_0 D(L) = D(L) \cap X_0$ to X_0 and we have

$$\sigma(L_0) = \{1\} \cup \left\{ \left(\frac{\pi k}{\varphi_0} \pm 1 \right)^2 : \ k \in \mathbb{N} \right\} \cup \left\{ \frac{\pi^2}{\varphi_0^2} k^2 : \ k \in \mathbb{N} \right\}. \tag{3.8}$$

Remark 3.4. By similar arguments we also could exclude the eigenspace corresponding to the eigenvalue 1. Observe that then the span of the two excluded spaces contains solenoidal fields which, however, we want to be included in the approach to the Stokes equations in Section 4.

By permanence properties of the $\mathcal{H}^\infty$-calculus this property remains valid for L_0, i.e., we have

$$L_0 \in \mathcal{H}^\infty(X_0), \quad \phi_{L_0}^\infty = 0. \tag{3.9}$$

The full linear operator related to (2.4)-(2.7) is now build up by an operator sum. We start by considering the operator $A := (\kappa + L_y)M$ in X_0 for fixed $\kappa > 0$ and with natural domain

$$D(A) := \{u \in D(M) : \ Mu \in D(L_y)\}.$$

Lemma 3.5. *The operator A defined above admits a bounded $\mathcal{H}^\infty$-calculus on X_0 with $\phi_A^\infty = 0$.*

Proof. Since L_y has a bounded $\mathcal{H}^\infty$-calculus on X_0 with $\phi_{L_y}^\infty = 0$ this remains true for the shifted operator $\kappa + L_y$. By the fact that X_0 has property (α), $M \in \mathcal{H}^\infty(X_0)$ with $\phi_M^\infty = 0$, and since $0 \in \rho(\kappa + L_y)$ for $\kappa > 0$, we may apply [17, Proposition 3.5] which yields the result. $\square$

Next, we consider $A+B$ with natural domain $D(A) \cap D(B)$. Since A and B are non-commuting (in the resolvent sense), here we employ a result obtained in [22, Theorem 3.1] which is based on the Labbas Terreni commutator condition, see (3.10) below.

Proposition 3.6. *Let E be a Banach space having property (α), $A, B \in \mathcal{H}^\infty(E)$ and $0 \in \rho(A)$. Further, assume that there are constants $C > 0$,*

$0 \le \alpha < \beta < 1$, $\psi_A > \phi_A^\infty$, $\psi_B > \phi_B^\infty$ satisfying $\psi_A + \psi_B < \pi$ and such that for all $\lambda \in \Sigma_{\pi-\psi_A}$ and all $\mu \in \Sigma_{\pi-\psi_B}$,

$$\|A(\lambda + A)^{-1}[A^{-1}, (\mu + B)^{-1}]\| \le \frac{C}{(1 + |\lambda|)^{1-\alpha}|\mu|^{1+\beta}}, \tag{3.10}$$

where $[A, B] = AB - BA$ denotes the commutator. Then there exists a $\nu > 0$ such that $\nu + A + B \in \mathcal{H}^\infty(E)$ with $\phi_{\nu+A+B}^\infty \le \max\{\psi_A, \psi_B\}$.

Remark 3.7. Notice that in [22] instead of property (α) for E the stronger property of an $\mathcal{R}$-bounded $\mathcal{H}^\infty$-calculus for B is assumed. However, in spaces having property (α) this is equivalent to having merely a bounded $\mathcal{H}^\infty$-calculus, see [12, Remark 12.10], [11].

Based on Proposition 3.6 we can prove

Lemma 3.8. *There is a $\nu > 0$ such that*

$$\nu + A + B \in \mathcal{H}^\infty(X_0), \quad \phi_{\nu+A+B}^\infty < \frac{\pi}{2}.$$

Proof. We compute the commutator in the Labbas Terreni condition for A and B. To this end, note that on the one hand it is clear that B and $L_y + \kappa$ are resolvent commuting, while on the other hand M and B do not commute. Instead, we have the relation

$$MBf = e^{2x}P(\partial_x)f = P(\partial_x - 2)e^{2x}f =: B_{-2}Mf \tag{3.11}$$

for all $f \in D(MB) := \{v \in D(B) : \ Bv \in D(M)\}$, satisfied in the sense of distributions, where at this point M and B are regarded as operators in $L^p(\mathbb{R})$. Note that $D(B_{-2}) = D(B)$. Now, fix $\eta > 0$ such that $\sigma(\eta + B) \cup \sigma(\eta + B_{-2}) \subset [\mathrm{Re} z > 0]$. For $\varphi \in C_c^\infty(\mathbb{R})$ we therefore obtain

$$\begin{aligned}
\big((\eta + B_{-2})^{-1}M(\eta + B)f, (\eta + B'_{-2})\varphi\big) &= (M(\eta + B)f, \varphi) \\
&= ((\eta + B_{-2})Mf, \varphi) \\
&= (Mf, (\eta + B'_{-2})\varphi).
\end{aligned}$$

Since $(\eta + B'_{-2})(C_c^\infty(\mathbb{R}))$ lies dense in $L_{p'}(\mathbb{R})$, where $1/p + 1/p' = 1$, this yields that $M(D(MB)) \subset D(B)$, hence $D(MB) \subset D(BM)$. Setting $f = (\mu + \eta + B)^{-1}g$ we arrive at

$$M(\mu + \eta + B)^{-1}g = (\mu + \eta + B_{-2})^{-1}Mg \quad (g \in D(M),\ \mu \in \Sigma_{\pi/2+\varepsilon}).$$

Regarded as operators in X_0 again, this results in

$$[(\omega + A), (\mu + \eta + B)^{-1}] = (\mu + \eta + B)^{-1}Q(\mu + \eta + B_{-2})^{-1}(\omega + A)$$

valid on $D(A)$ for all $\mu \in \Sigma_{\pi/2+\varepsilon}$, $\omega > 0$, and with the first order differential operator

$$Q = B - B_{-2} = P(\partial_x) - P(\partial_x - 2) = Q(\partial_x) = -4\partial_x - 4\beta + 4.$$

Thus, we deduce

$$(\omega + A)(\lambda + \omega + A)^{-1}[(\omega + A)^{-1}, (\mu + \eta + B)^{-1}]$$

$$= -(\lambda+\omega+A)^{-1}(\mu+\eta+B)^{-1}Q(\mu+\eta+B_{-2})^{-1}$$

which implies

$$\begin{aligned}&\|(\omega+A)(\lambda+\omega+A)^{-1}[(\omega+A)^{-1},(\mu+\eta+B)^{-1}]\|\\ &\leq \frac{C}{(1+|\lambda|)|\mu|^{3/2}}\end{aligned}$$

for all $\lambda \in \Sigma_{\pi-\delta}$ and $\mu \in \Sigma_{\pi/2+\varepsilon}$ for fixed $\varepsilon \in (0,\pi/2-\phi_B^\infty)$ and $\delta \in (0,\pi/2-\varepsilon)$. Hence, the Labbas Terreni condition (3.10) holds true with $\alpha = 0$, $\beta = 1/2$, $\psi_A = \delta$ and $\psi_B = \pi/2-\varepsilon$. Applying Proposition 3.6 in combination with Lemma 3.5 the assertion follows. □

The next Proposition is crucial in our approach, since it gives a sufficient condition for the invertibility of an operator sum without requiring a shift. It is due to Prüß, cf. [20, Theorem 8.5]. We also remark that the class $\mathcal{BIP}(X)$ appearing in the statement of the proposition below contains the class $\mathcal{H}^\infty(X)$, cf. [3], [11], [12]. Hence it applies to our situation.

Proposition 3.9. *Suppose the Banach space E belongs to the class $\mathcal{HT}$ and assume*

1. *$\omega_A + A$, $\omega_B + B \in \mathcal{BIP}(E)$ for some $\omega_A, \omega_B \in \mathbb{R}$;*
2. *A and B are resolvent commuting;*
3. *$\theta_{A+\omega_A} + \theta_{B+\omega_B} < \pi$.*

Then $A+B$ with domain $D(A+B) = D(A)\cap D(B)$ is closed and $\sigma(A+B) \subset \sigma(A)+\sigma(B)$. In particular, if $\sigma(A)\cap\sigma(-B) = \emptyset$ then $A+B$ is invertible.

Applying this result to $A+B$ and L_0 leads to

Lemma 3.10. *Let the operator $A+B+L_0$ in X_0 with natural domain $D(A+B)\cap D(L_0)$ be defined as above. Furthermore, let $\lambda_1 > 0$, being the first eigenvalue of L_0 (see (3.8)), satisfy*

$$\lambda_1 > \beta^2 = \left(2 - \frac{2}{p} - \frac{\gamma}{p}\right)^2. \tag{3.12}$$

Then $A+B+L_0$ is invertible.

Proof. By Lemma 3.8 and (3.9) conditions (1), (2) and (3) of Proposition 3.9 are readily fulfilled. Hence, $A+B+L_0$ is closed. Next, $A+B+\beta^2$ with domain $D(A)\cap D(B)$ is accretive. This can be seen by elementary showing

$$((A+B+\beta^2)u, u|u|^{p-2})_{p,p',\mathbb{R}^2} \geq 0 \quad (u \in D(A)\cap D(B)).$$

As a consequence $\sigma(A+B) \subset \{\mathrm{Re} z \geq -\beta^2\}$. Thus, if condition (3.12) is satisfied, we have $\sigma(A+B)\cap\sigma(-L_0) = \emptyset$ and Proposition 3.9 yields the assertion. □

Note that $A+B+L_0$ represents the full linear operator of the transformed problem (2.4)-(2.7). Combining Lemma 3.8 and Lemma 3.10 leads to

Proposition 3.11. *Let condition (3.12) be satisfied. Then we have*

$$A + B + L_0 \in \mathcal{H}^\infty(X_0), \quad \phi^\infty_{A+B+L_0} < \frac{\pi}{2}.$$

Proof. For simplicity set $T = A + B + L_0$. In view of Lemma 3.8 we know that $\phi^\infty_{\nu+A+B} < \pi/2$ and by the discussion before also that $\phi^\infty_{L_0} = 0$. Due to the fact that $\nu + A + B$ and L_0 are resolvent commuting the standard Kalton-Weis theorem, cf. [11, Theorem 4.4] (see also [17, Proposition 3.5]), therefore implies $\nu + T \in \mathcal{H}^\infty(X_0)$ and $\phi^\infty_{\nu+T} < \pi/2$. Now, fix $\phi \in (\phi^\infty_{\nu+T}, \pi)$ and let for $\theta \in (\phi^\infty_{\nu+T}, \phi)$ the path Γ be given as

$$\Gamma := \{te^{i\theta} : \infty > t > 0\} \cup \{te^{-i\theta} : 0 \le t < \infty\}.$$

Then for $h \in \mathcal{H}_0(\Sigma_\phi)$ we have to estimate the Dunford integral

$$h(T) = \frac{1}{2\pi i} \int_\Gamma h(\lambda)(\lambda - T)^{-1} d\lambda. \tag{3.13}$$

If we split this integral into two parts corresponding either to $|\lambda| \le 1$ or to $|\lambda| > 1$, then the desired estimate for small λ easily follows from $0 \in \rho(T)$ which has been proved in Lemma 3.10. On the other hand, the part corresponding to $|\lambda| > 1$ easily reduces to $\nu + T \in \mathcal{H}^\infty(X_0)$ which has been derived above. Hence the assertion is proved. □

Now we are in position to rigorously prove equivalence of problems (1.2) and (2.4)-(2.7). To this end, recall that the domain of $A + B + L_0$ by the results obtained above is given as

$$\begin{aligned} &D(A + B + L_0) \\ &= \Big\{ v = (v_x, v_\varphi, v_y) \in X_0 : \ e^{2x} v \in L^p(\mathbb{R}_x \times I_\varphi, W^{2,p}(\mathbb{R}_y, \mathbb{R}^3)), \\ &\quad v \in L^p(\mathbb{R}_y, W^{2,p}(\mathbb{R}_x \times I_\varphi, \mathbb{R}^3)),\ \partial_\varphi v_x = v_\varphi = \partial_\varphi v_y = 0 \text{ on } \partial\Omega \Big\}. \end{aligned} \tag{3.14}$$

Let $1 < p < \infty$, $\gamma \in \mathbb{R}$, and $\varphi_0 \in (0, \pi)$ be given such that condition (3.12) is satisfied. Let Θ^* be the pull back defined in (2.1) and $\tilde{\Theta}^*$ be the transformation given in (2.2). It is clear that by construction

$$\tilde{\Theta}^* : L^p_\gamma(G, \mathbb{R}^3) \to L^p(\Omega, \mathbb{R}^3)$$

is an isomorphism with inverse $\tilde{\Theta}_* = \Theta_* e^{-2x}$. Utilizing decomposition (3.7) we see that

$$L^p_\gamma(G, \mathbb{R}^3) = \tilde{\Theta}_* X_0 \oplus \tilde{\Theta}_* L^p(\mathbb{R}^2, E_0), \tag{3.15}$$

hence that also

$$\tilde{\Theta}^* : \tilde{\Theta}_* X_0 \to X_0$$

is an isomorphism with X_0 defined in (3.5).

Observe that by the discussion in Section 2 we also have

$$\tilde{\Theta}^*(\kappa - \Delta)u = \tilde{\Theta}^* f = g = (A + B + L_0)\Theta^* u. \tag{3.16}$$

Thus, we can define

$$\mathcal{A}_\kappa u := (\kappa - \Delta)u, \quad u \in D(\mathcal{A}_\kappa) := \Theta_* D(A + B + L_0), \tag{3.17}$$

which is an operator in $\tilde{\Theta}_* X_0$. By the transforms calculated in Section 2 it is straight forward to show that $D(\mathcal{A}_\kappa)$ is explicitly given as

$$\begin{aligned} D(\mathcal{A}_\kappa) = \Big\{ & u \in \tilde{\Theta}_* X_0 : \ u/|(\cdot,\cdot,0)|^2,\ \partial^\alpha u \in L^p_\gamma(G, \mathbb{R}^3)\ (|\alpha| \le 2), \\ & \nu \times \operatorname{curl} u = 0,\ \nu \cdot u = 0 \text{ on } \partial G \Big\}. \end{aligned} \tag{3.18}$$

Summarizing, we have proved

Lemma 3.12. *Let $1 < p < \infty$, $\gamma \in \mathbb{R}$, and $\varphi_0 \in (0, \pi)$ be given such that condition (3.12) is satisfied. Assume that $f \in \tilde{\Theta}_* X_0$ and $g = \tilde{\Theta}^* f$. Then $v \in D(A + B + L_0)$ (given through (3.14)) is the unique solution of (2.4)-(2.7) if and only if $u = \Theta_* v \in D(\mathcal{A}_\kappa)$ (given through (3.18)) is the unique solution of (1.2). In particular, $\tilde{\Theta}^* \in \mathscr{L}_{is}(\tilde{\Theta}_* X_0, X_0)$ and $\Theta^* \in \mathscr{L}_{is}(D(\mathcal{A}_\kappa), D(A + B + L_0))$.*

By the fact that the property of having an $\mathcal{H}^\infty$-calculus is invariant under conjugation with isomorphisms we obtain the following result.

Proposition 3.13. *For $\kappa > 0$ let $\mathcal{A}_\kappa$ be defined as above. Let $1 < p < \infty$, $\gamma \in \mathbb{R}$, and $\varphi_0 \in (0, \pi)$ be given such that condition (3.12) is satisfied. Then we have*

$$\mathcal{A}_\kappa \in \mathcal{H}^\infty(\tilde{\Theta}_*(X_0)), \quad \phi^\infty_{\mathcal{A}_\kappa} < \frac{\pi}{2}.$$

Proof. Note that due to Cauchy's theorem, formula (3.1) can be reformulated as

$$h(T) = \frac{1}{2\pi i} \int_\Gamma \frac{h(\lambda)}{\lambda} T(\lambda - T)^{-1} d\lambda \tag{3.19}$$

for $h \in \mathcal{H}_0(\Sigma_\phi)$, $T \in \mathcal{S}(X)$, and $\phi \in (\phi_T, \pi)$. The fact that $T \in \mathcal{H}^\infty(X)$ then means that

$$\|h(T)\|_{\mathscr{L}(X)} \le C \|h\|_\infty \quad (h \in \mathcal{H}_0(\Sigma_\phi)) \tag{3.20}$$

for $h(T)$ given through (3.19). By Proposition 3.11 this is true for $T = A + B + L_0$ and $\phi \in (\phi^\infty_{A+B+L_0}, \pi)$.

Now, observe that in view of (3.16) we have

$$\mathcal{A}_\kappa u = \tilde{\Theta}_*(A + B + L_0)\Theta^* u \quad (u \in D(\mathcal{A}_\kappa)).$$

Thanks to Lemma 3.12 this yields

$$\mathcal{A}_\kappa(\lambda - \mathcal{A}_\kappa)^{-1} = \tilde{\Theta}_*(A + B + L_0)(\lambda - (A + B + L_0))^{-1}\tilde{\Theta}^*$$

for $\lambda \in \rho(\mathcal{A}_\kappa) = \rho(A+B+L_0)$. By this representation and formula (3.19) we easily achieve that (3.20) remains valid for $T = \mathcal{A}_\kappa$ and $\phi \in (\phi^\infty_{A+B+L_0}, \pi)$. Hence the assertion is proved. $\square$

Since an $\mathcal{H}^\infty$-calculus implies maximal regularity we also have

Corollary 3.14. *Suppose the assumptions of Proposition 3.13 hold and let $J = (0,T)$ with $T \in (0,\infty)$. Then for each $f \in L^p(J, \tilde{\Theta}_* X_0)$ there exists a unique solution $u \in L^p(J, \tilde{\Theta}_* X_0)$ of (1.2) such that*

$$u/|(x_1,x_2)|^2, \partial_t u, \nabla^2 u \in L^p(J, L^p_\gamma(G, \mathbb{R}^3)).$$

In particular, the map $[u \mapsto f]$ defines an isomorphism between the corresponding spaces.

Before turning to the Stokes equations let us have a closer look at the essential condition (3.12). Especially we are interested when it is allowed to choose $\gamma = 0$, that is when we can work in the unweighted setting. The relationship on the first eigenvalue λ_1 of L_0 can be written as

$$(2 - \sqrt{\lambda_1})p - 2 < \gamma < (2 + \sqrt{\lambda_1})p - 2,$$

Since $\lambda_1 = \min\{1, (\frac{\pi}{\varphi_0} - 1)^2\}$, we have a closer look at the condition

$$\left(3 - \frac{\pi}{\varphi_0}\right) p - 2 < \gamma < \left(1 + \frac{\pi}{\varphi_0}\right) p - 2 \tag{3.21}$$

in terms of $\gamma \in \mathbb{R}$, $p \in (1,\infty)$ and the angle $\varphi_0 \in (0,\pi)$. The following tabular displays γ-intervals for some characteristic angles φ_0.

φ_0	$\gamma \in$	$\gamma = 0 : p \in$
$\varphi_0 \leq \frac{\pi}{2}$	$(p-2, 3p-2)$	$(1,2)$
$\varphi_0 = \frac{3}{4}\pi$	$(\frac{5}{3}p - 2, \frac{7}{3}p - 2)$	$(1, \frac{6}{5})$
$\varphi_0 = (1-\varepsilon)\pi$	$((3 - 1/(1-\varepsilon))p - 2, (1 + 1/(1-\varepsilon))p - 2)$	$\left(1, \frac{2(1-\varepsilon)}{3-1}\right)$

In terms of condition (3.12) the answer to the above question is illustrated in the last column of the table. However, by duality and interpolation we even deduce that for each angle $\varphi_0 \in (0,\pi)$ and $\gamma = 0$ the full range $1 < p < \infty$ is available. We establish this observation as

Corollary 3.15. *Let $1 < p < \infty$, $\varphi_0 \in (0,\pi)$ and set $\gamma = 0$. Then the assertion of Proposition 3.13 for $\mathcal{A}_\kappa$ still hold true (on the domain $D(\mathcal{A}_\kappa)$ canonically defined by duality and interpolation). Hence $\mathcal{A}_\kappa$ has also maximal regularity on $\tilde{\Theta}_*(X_0)$.*

Proof. For $p = 2$ the operator $\mathcal{A}_\kappa$ is selfadjoint. For the time being we indicate the p-dependence of the base space X_0, i.e., we write X_0^p. Since $(L^p(\Omega))_{1<p<\infty}$ is an interpolation scale, e.g. for the real method, also $(X_0^p)_{1<p<\infty}$ and hence also $(\tilde{\Theta}_* X_0^p)_{1<p<\infty}$ is an interpolation scale. Since the dual space of X_0^p is represented as $(X_0^p)' = X_0^{p'}$, we also have $(\tilde{\Theta}_* X_0^p)' = \tilde{\Theta}_* X_0^{p'}$ for $1/p + 1/p' = 1$. But then, since for any angle $\varphi_0 \in (0,\pi)$ the assertions hold at least on a small interval $p \in (1, \varepsilon)$, the general case easily follows by standard duality and interpolation arguments. $\square$

Remark 3.16. Let us compare the situation here to some known conditions on the weight γ for the heat equation in a wedge. Nazarov discussed the case of Dirichlet and Neumann boundary conditions in [18]. In the special case of a three-dimensional wedge Nazarovs' conditions take the form

$$2 - \frac{2}{p} - \lambda_D < \frac{\gamma}{p} < 2 - \frac{2}{p} + \lambda_D$$

for a Dirichlet boundary condition and

$$2 - \frac{2}{p} - \min\{\lambda_N, 2\} < \frac{\gamma}{p} < 2 - \frac{2}{p}$$

for a Neumann boundary condition. Here $\lambda_D = \lambda_N = \pi/\varphi_0$ denote the square roots of the first nonnegative eigenvalues of the related azimuthal operators which corresponding to L in this work.

Thus, in the situations considered in [18] the admissible range for γ is larger than the range for perfect slip obtained by condition (3.12). We remark, however, that for the problem considered in this work the form of the first eigenvalue $\lambda_1 = \min\{1, (\pi/\varphi_0 - 1)^2\}$ in (3.12) is due to the fact that we have to transform a system including vector fields. We also remark that by excluding the eigenspace corresponding to the eigenvalue 1 of L (see (3.3)) our condition would improve in case that $\varphi_0 < \pi/2$. Then, however, we miss some solenoidal functions, see also Remark 3.4. On the other hand, including the eigenspace corresponding to the eigenvalue 0 would cause our approach to fail, since then the condition $\sigma(A+B) \cap \sigma(-L) = \emptyset$ (see proof of Lemma 3.10) cannot be satisfied anymore.

4. The Stokes equations on a wedge

We turn to the Stokes equations (1.4). To this end, we first have to fix a suitable space of solenoidal vector fields. Let $1 < p < \infty$ and $1/p + 1/p' = 1$. In our setting it seems appropriate to choose

$$L^p_{\sigma,\gamma}(G) := \left\{ u \in L^p_\gamma(G, \mathbb{R}^3) : \int_G u \cdot \nabla\varphi d(x_1, x_2, y) = 0 \ \left(\varphi \in \widehat{W}^{1,p'}_{-\gamma'}(G)\right) \right\},$$

where $\gamma' = \gamma p'/p$ and

$$\widehat{W}^{1,p'}_{-\gamma'}(G) := \{\varphi \in L^1_{loc}(G) : \ \nabla\varphi \in L^{p'}_{-\gamma'}(G)\}$$

and where $u \in L^1_{loc}(G)$ means that u is integrable on every compact $K \subset G$. Since $C_c^\infty(G) \subset \widehat{W}^{1,p}_{-\gamma}(G)$, it is obvious that $u \in L^p_{\sigma,\gamma}(G)$ satisfies $\operatorname{div} u = 0$ in the sense of distributions. Moreover, by the generalized Gauß theorem, cf. [8, Theorem III.2.2], the trace $\nu \cdot u$ is welldefined in the trace space (Slobodeckii space) $W_p^{-1/p}(\mathcal{O})$ for every bounded domain $\mathcal{O}$ such that $\overline{\mathcal{O}} \subset \partial G \setminus (\{(0,0)\} \times \mathbb{R})$. Hence $u \cdot \nu = 0$ on ∂G is fulfilled at least in a local sense away from 0. Our intention is to regard the Stokes operator as the part of the Laplacian in the space $\tilde{\Theta}_* X_0$. For this purpose, we first need to show

Lemma 4.1. *There is a canonical embedding*

$$L^p_{\sigma,\gamma}(G) \hookrightarrow \tilde{\Theta}_* X_0, \tag{4.1}$$

that is, $L^p_{\sigma,\gamma}(G)$ can be regarded as a closed subspace of $\tilde{\Theta}_* X_0$.

Proof. Consider the factor space

$$Y := L^p_\gamma(G, \mathbb{R}^3)/\tilde{\Theta}_* L^p(\mathbb{R}^2, E_0)$$

with E_0 defined in (3.6). Recall that an element of $L^p(\mathbb{R}^2, E_0)$ is represented by $(0, 0, w)$ with $w \in L^p(\mathbb{R}^2)$. Applying the transformed divergence operator (see (2.3)) to $(0, 0, w)$ yields $\partial_y w = 0$. Thus w is constant in y which results in $w = 0$. This implies $L^p_{\sigma,\gamma}(G) \cap \tilde{\Theta}_* L^p(\mathbb{R}^2, E_0) = \{0\}$, hence that

$$L^p_{\sigma,\gamma}(G) \hookrightarrow Y$$

From decomposition (3.15) we infer that Y is isomorphic to $\tilde{\Theta}_* X_0$ (with respect to the L^p_γ-norm), hence embedding (4.1) is well-defined in a canonical way. Since $L^p_{\sigma,\gamma}(G)$ and $\tilde{\Theta}_* X_0$ are obviously closed with respect to the norm in $L^p_\gamma(G, \mathbb{R}^3)$, the claim is proved. $\square$

Remark 4.2. Observe that the embedding operator which maps $L^p_{\sigma,\gamma}(G)$ isomorphically onto a closed subspace of $\tilde{\Theta}_* X_0$ is represented by $\tilde{\Theta}_* \Pi_0 \tilde{\Theta}^*$ with Π_0 defined in (3.4). Hence we identify $L^p_{\sigma,\gamma}(G)$ actually with $\tilde{\Theta}_* \Pi_0 \tilde{\Theta}^*(L^p_{\sigma,\gamma}(G))$. However, the fact that $\tilde{\Theta}^*(\kappa - \Delta)\Theta_* = A + B + L$ commutes with Π_0 justifies it to work directly with $L^p_{\sigma,\gamma}(G)$ in the set up of the Stokes operator, as it is presented below.

Let $\mathcal{A}_\kappa : D(\mathcal{A}_\kappa) \subset \tilde{\Theta}_* X_0 \to \tilde{\Theta}_* X_0$ be the Laplacian as defined in (3.17) with domain $D(\mathcal{A}_\kappa)$ as given in (3.18). We also set $\mathcal{A} := \mathcal{A}_0$, i.e. for $\kappa = 0$. Thanks to Lemma 4.1 (and Remark 4.2) we can define the Stokes operator as the part of $\mathcal{A}$ in $L^p_{\sigma,\gamma}(G)$, that is, we set

$$\begin{aligned} \mathcal{A}_S u &:= \mathcal{A}|_{L^p_{\sigma,\gamma}(G)} u, \quad u \in D(\mathcal{A}_S), \\ D(\mathcal{A}_S) &:= \{v \in D(\mathcal{A}) \cap L^p_{\sigma,\gamma}(G) : \ \mathcal{A}v \in L^p_{\sigma,\gamma}(G)\}, \end{aligned}$$

Note that then (1.4) is equivalent to the Cauchy problem

$$\left\{ \begin{aligned} u' + \mathcal{A}_S\, u &= f, \quad t \in (0, T), \\ u(0) &= 0, \end{aligned} \right. \tag{4.2}$$

with $f \in L^p_{\sigma,\gamma}(G)$. The following lemma justifies the above definition of the Stokes operator.

Lemma 4.3. *We have*

$$D(\mathcal{A}_S) = D(\mathcal{A}) \cap L^p_{\sigma,\gamma}(G).$$

Proof. We only have to show, that the right hand side is a subset of $D(\mathcal{A}_S)$. To this end, let $u \in D(\mathcal{A}) \cap L^p_{\sigma,\gamma}(G)$. Then there exist $f \in L^p_\gamma(G,\mathbb{R}^3)$ and $\lambda \in \rho(\mathcal{A})$ such that $u = (\lambda - \mathcal{A})^{-1} f$. Since the resolvents of $\mathcal{A}$ and $\mathcal{A}_S$ in particular fulfill

$$(\lambda - \mathcal{A}_S)^{-1} = (\lambda - \mathcal{A})^{-1}|_{L^p_{\sigma,\gamma}(G)}$$

we readily obtain $u = (\lambda - \mathcal{A}_S)^{-1} f \in D(\mathcal{A}_S)$ provided we can show that $f \in L^p_{\sigma,\gamma}(G)$. By the fact that

$$f = (\lambda - \mathcal{A})u = (\lambda - \operatorname{curl}^2)u$$

and $u \in D(\mathcal{A}) \cap L^p_{\sigma,\gamma}(G)$ the Gauß theorem yields

$$\begin{aligned} \int_G f \cdot \nabla\varphi \, d(x_1,x_2,y) &= -\int_G (\operatorname{curl}\operatorname{curl} u) \cdot \nabla\varphi d(x_1,x_2,y) \\ &= -\int_{\partial G} (\nu \times \operatorname{curl} u) \cdot \nabla\varphi d\sigma = 0 \end{aligned}$$

for all $\varphi \in C^\infty\left(\overline{G} \setminus (\{(0,0)\} \times \mathbb{R})\right) \cap \widehat{W}^{1,p'}_{-\gamma'}(G)$. This implies $f \in L^p_{\sigma,\gamma}(G)$, hence $u \in D(\mathcal{A}_S)$, provided $C^\infty(\overline{G} \setminus (\{(0,0)\} \times \mathbb{R})) \cap \widehat{W}^{1,p'}_{-\gamma'}(G)$ lies dense in $\widehat{W}^{1,p'}_{-\gamma'}(G)$. To see this, observe that it is not difficult to construct a bi-Lipschitz map from G to $\mathbb{R}^3$ which is singular only on $\{(0,0)\}\times\mathbb{R}$ and smooth otherwise. Thus $\widehat{W}^{1,p'}_{-\gamma'}(G)$ and $\widehat{W}^{1,p'}_{-\gamma'}(\mathbb{R}^3)$ are isomorphic. The assertion for $\mathbb{R}^3$, however, can easily be obtained by a mollifier argument. The properties of the bi-Lipschitz map then yield the desired density. □

Remark 4.4. For $\gamma = 0$ we can work with the Helmholtz projection $\mathbb{P}$ as usually. It is given by

$$\mathbb{P} : L^p(G,\mathbb{R}^3) \to L^p_\sigma(G), \quad u \mapsto u - \nabla p,$$

where p is the solution of the weak Neumann problem

$$(\nabla p, \nabla\varphi) = (u, \nabla\varphi) \quad (\varphi \in \widehat{W}^{1,p'}(G)),$$

for $u \in L^p(G,\mathbb{R}^3)$. We refer to [14] for the existence of the Helmholtz decomposition of $L^p(G,\mathbb{R}^3)$, $1 < p < \infty$. Note also that in this case we have $L^p_\sigma(G) = \overline{C^\infty_{c,\sigma}(G)}^{L^p}$. With this projection at hand the Stokes operator takes the form

$$\mathcal{A}_S u = \mathbb{P}\mathcal{A}u = -\mathbb{P}\Delta u, \quad u \in D(\mathcal{A}_S). \tag{4.3}$$

This representation will be utilized in the next section.

By the discussion above we have the relation

$$(\lambda - \mathcal{A}_S)^{-1} = (\lambda - \mathcal{A})^{-1}|_{L^p_{\sigma,\gamma}(G)}, \tag{4.4}$$

for $\lambda \in \rho(\mathcal{A}) = \rho(\mathcal{A}_S)$. Proposition 3.13 and Corollary 3.15 therefore immediately imply

Proposition 4.5. *For $\kappa > 0$ let $\mathcal{A}_{S,\kappa} := \kappa + \mathcal{A}_S$ with $\mathcal{A}_S$ the Stokes operator as defined above. Let either $1 < p < \infty$, $\gamma \in \mathbb{R}$, and $\varphi_0 \in (0, \pi)$ be given such that condition (3.12) is satisfied or let $\gamma = 0$ and $\varphi_0 \in (0, \pi)$, $1 < p < \infty$ be arbitrary. Then we have*

$$\mathcal{A}_{S,\kappa} \in \mathcal{H}^\infty(L^p_{\sigma,\gamma}(G)), \quad \phi^\infty_{\mathcal{A}_{S,\kappa}} < \frac{\pi}{2}.$$

Remark 4.6. Applying the scaling argument utilized in the next section to the $\mathcal{H}^\infty$ estimate for $\mathcal{A}_{S,\kappa}$ yields that Proposition 4.5 also holds for $\kappa = 0$. This, of course, is also true for Proposition 3.13 and essentially relies on the fact that a wedge is scaling invariant.

Note that Proposition 4.5 and Remark 4.6 imply Theorem 1.1, our main result for the linearized situation.

5. The Navier-Stokes equations

Here we consider the non-linear Navier-Stokes equations (1.1) on the three-dimensional wedge G. For simplicity we restrict ourselves to the case $\gamma = 0$, i.e. to the unweighted setting. We will apply the abstract result [9, Theorem 1] in order to derive mild solvability.

Note that by Theorem 1.1 we know that $\mathcal{A}_S$ generates a bounded holomorphic C_0-semigroup $(e^{-t\mathcal{A}_S})_{t\geq 0}$. Following the setting in [9] we write the nonlinearity as $\mathbb{P}(u \cdot \nabla)u = \sum_{j=1}^3 \Gamma_j G_j(u)$ with $\Gamma_j u = \mathbb{P}\partial_j u$ and $G_j(u) = u^j u$. We prove Theorem 1.4 by varifying the conditions (A), $(N1)$, and $(N2)$ in [9] which, adapted to our situation, read as follows:

(A) The estimate

$$\|e^{-t\mathcal{A}_S} u_0\|_p \leq M \frac{\|u_0\|_s}{t^\sigma} \quad (u_0 \in L^s_\sigma(G),\ 0 < t < \infty) \tag{5.1}$$

holds with $\sigma = \frac{3}{2}(\frac{1}{s} - \frac{1}{p})$, $p \geq s > 1$ and a constant M depending only on p, s.

$(N1)$ The estimate

$$\|e^{-t\mathcal{A}_S} \sum_{j=1}^3 \Gamma_j u_0\|_p \leq N_1 \frac{\|u_0\|_p}{t^{1/2}} \quad (u_0 \in C^\infty_{c,\sigma}(G),\ 0 < t < \infty) \tag{5.2}$$

holds with N_1 depending only on $p \in (1, \infty)$.

$(N2)$ For the nonlinear terms $G_j(u)$ we have $G_j(0) = 0$ and the estimate

$$\|G_j(v) - G_j(w)\|_s \leq N_2 \|v - w\|_p (\|v\|_p + \|w\|_p) \quad (j = 1, 2, 3) \tag{5.3}$$

with $1 \leq s = \frac{p}{2}$ and N_2 depending only on p for $1 < p < \infty$.

It is obvious that in our case $(N2)$ is satisfied. In order to show (A) and $(N1)$ we require the equivalence of $\|\nabla^k \cdot\|_p$ and $\|\mathcal{A}_S^{k/2} \cdot\|_p$ for $k = 1, 2$. This can be achieved by employing a standard scaling argument which is utilized in e.g. [1], [19], [23]. Mostly it is applied in a half-space setting, but of course it applies to any scaling invariant domain, hence also to wedges. Let $\lambda > 0$ and $k = 1, 2$. Due to $\mathcal{A}_S \in \mathcal{H}^\infty(L^p_\sigma(G))$ we obtain the equivalence of norms

$$\|w\|_{k,p} \sim \|(1+\mathcal{A}_S)^{k/2} w\|_p \quad \Big(w \in D(\mathcal{A}_S^{k/2})\Big)$$

for $w(x) := u(\lambda x)$ and therefore

$$\frac{1}{\lambda^k}\|u\|_p + \|\nabla^k u\|_p \sim \|(\frac{1}{\lambda^2} + \mathcal{A}_S)^{k/2} u\|_p \quad \Big(u \in D(\mathcal{A}_S^{k/2})\Big).$$

Taking the limit $\lambda \to \infty$ we have

$$\|\nabla^k u\|_p \sim \|\mathcal{A}_S^{k/2} u\|_p \quad \Big(u \in D(\mathcal{A}_S^{k/2})\Big). \tag{5.4}$$

In particular, utilizing

$$\|\nabla^2 u\|_p \leq c\|\mathcal{A}_S u\|_p \quad (u \in D(\mathcal{A}_S))$$

for a $c > 0$, condition (A) follows by standard arguments relying on the Gagliardo-Nirenberg inequality. This is well-known and explicitely shown e.g. in Proposition 3.1 in [24]. Note that the Gagliardo-Nirenberg inequality holds true on (ε, ∞) domains, cf. [23] and [6, Theorem 9.3]. By a general result of Jones in [10] a domain is an (ε, ∞) domain if and only if there is a bi-Lipschitz mapping to an (ε, ∞) domain. For a wedge such a mapping to the halfspace, which is known to be an (ε, ∞) domain, can be easily constructed using a suitable piecewise linear function.

Let us consider condition $(N1)$. We estimate

$$\begin{aligned} \|e^{-t\mathcal{A}_S}\sum_{j=1}^3 \mathbb{P}\partial_j u_0\|_p &= \|\mathcal{A}_S^{1/2} e^{-t\mathcal{A}_S}\mathcal{A}_S^{-1/2}\sum_{j=1}^3 \mathbb{P}\partial_j u_0\|_p \\ &\leq \frac{c}{t^{1/2}}\|\mathcal{A}_S^{-1/2}\sum_{j=1}^3 \mathbb{P}\partial_j u_0\|_p \\ &\leq \frac{c}{t^{1/2}}\|u_0\|_p, \qquad (u_0 \in C^\infty_{c,\sigma}(G)), \end{aligned}$$

where in the second step we make use of the fact, that $\mathcal{A}_S$ is the generator of a bounded holomorphic C_0-semigroup and in the last step we use the boundedness of

$$\mathcal{A}_S^{-1/2}\mathbb{P}\partial_j : L^p_\sigma(G) \to L^p_\sigma(G)$$

for $j = 1, 2, 3$. This follows from (5.4) for $k = 1$ and a duality argument by the fact that

$$(\mathcal{A}_S^{-1/2}\mathbb{P}\partial_j u, v) = -(u, \partial_j \mathcal{A}_S^{-1/2} v) \quad (u \in C^\infty_{c,\sigma}(G),\ v \in L^{p'}_\sigma(G)).$$

Hence $(N1)$ is fulfilled and Theorem 1.4 is proved.

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