Intersection Theory

Oberseminar Algebra, Geometrie und Zahlentheorie

Winter 2019/20

The aim of the seminar is to complement Jens Hornbostel's GRK lectures on Chow rings (and other topics) with concrete calculations and applications. As such, all talks will concentrate on examples rather than general theory.

Many talks address one particular question, marked in blue below. The answer should preferably be presented in top-down style: After explaining the question, provide first only a rough idea of how to approach it. Then give a more detailed overview over the argument/the computation that leads to an answer. Finally, fill in selected details. A rigorous proof will frequently depend on deep theorems that have not (yet) been touched upon in the lectures, yet alone in the seminar. State these theorems clearly, but do not feel obliged to comment on the proofs.

The seminar is based on the following outstanding monograph. All section and theorem numbers below refer to this source. We will concentrate on chapters 2, 3, 4 and 6, consulting the more "theoretical" chapters 1 and 5 only as necessary as we go along.

[EH16] D. Eisenbud and J. Harris, *3264 and All That: A Second Course in Algebraic Geometry*, Cambridge University Press, Cambridge, 2016.

Five copies are available from the Zentralbibliothek (`mat d 137` in Lehrbuchsammlung). An electronic copy can be obtained from the second author's homepage:

`https://scholar.harvard.edu/files/joeharris/files/000-final-3264.pdf`

First examples

- **1** (11.10.) Bézout's Theorem and the Circles of Appollonius
- **2** (18.10.) Dual varieties and plane counting
- **3** (25.10.) The class of a graph and triples of polynomials

The geometry of Grassmannians

- **4** (15.11.) Introduction to Grassmannians
- **5** (22.11.) The Chow ring of the Grassmannian of lines in three-space
- **6** (29.11.) Lines meeting things
- **7** (06.12.) Schubert cells
- **8** (13.12.) The Chow ring of Grassmannians
- **9** (20.12.) The degree of a surface swept out by a twisted cubic

Fano schemes and the twenty-seven lines on a cubic surface

- **10** (10.01.) Fano schemes and the existence of lines in a cubic surface
- **11** (17.01.) Tangent spaces to Fano schemes
- **12** (24.01.) The exact number of lines in a cubic surface

First examples

Talk 1: Bézout's Theorem and the Circles of Appollonius

11.10., Marcus Zibrowius

Introduce all concepts necessary to state and understand Bézout's Theorem (§ 2.1.1, Corollary 2.4). State the theorem and explain how to prove it use earlier theorems about the Chow ring. Sketch how Bézout's Theorem can be applied to answer the following question (§ 2.2): How many circles are tangent to three given circles?

Talk 2: Dual varieties and plane counting

18.10., Herman Rohrbach

Answer the following question, following § 2.1.3: Consider a smooth cubic surface S and a line L in $\mathbb{P}^3$. How many planes containing L are tangent to S?

Talk 3: The class of a graph and triples of polynomials

25.10., Johannes Fischer

Answer the following question, following § 2.1.7: Let A, B and C be homogeneous polynomials of the same degree in three variables. For how many triples $\mathbf{t} = (t_0, t_1, t_2)$ is $(A(\mathbf{t}), B(\mathbf{t}), C(\mathbf{t}))$ a scalar multiple of $\mathbf{t}$?

The geometry of Grassmannians

Talk 4: Introduction to Grassmannians

15.11., Moritz Petschick

Present § 3.1 and give an overview of § 3.2, with an emphasis on geometric intuition. Zoom in on one result of § 3.2 of your choice.

Talk 5: The Chow ring of the Grassmannian of lines in three-space

22.11., David Bradley-Williams

Present § 3.3. This is a hands-on computation that should provide a very concrete illustration of the general theory of Chow rings.

Talk 6: Lines meeting things

29.11., Leif Zimmermann

Answer the following two questions about lines in $\mathbb{P}^3$, following § 3.4: How many lines meet four given lines? How many lines meet a curve?

Talk 7: Schubert cells

06.12., Benjamin Klopsch

Give an overview of the definitions and results of § 4.1. Make sure to illustrate each statement with a low-dimensional example.

Talk 8: The Chow ring of Grassmannians

13.12., Immi Halupczok

Present as much of §§ 4.2.1 and 4.2.2 as is reasonable in one talk.

Talk 9: The degree of a surface swept out by a twisted cubic

20.12., Heng Xie

Answer the following question, following § 4.2: Let $C \subset \mathbb{P}^5$ be a twisted cubic curve that is contained in the Grassmannian $\mathbb{G}(1, 3) \subset \mathbb{P}^5$. What is the degree of the surface $S \subset \mathbb{P}^3$ swept out by the lines corresponding to the points of C?

Fano schemes and the twenty-seven lines on a cubic surface

Talk 10: Fano schemes and the existence of lines in a cubic surface

10.01., Thuong Dang

Present §§ 6.1 and 6.2. In particular, show that every cubic surface contains lines (cf. § 5.1 and end of § 6.2).

Talk 11: Tangent spaces to Fano schemes

17.01., Jakob Bergqvist

Present Theorem 6.13 and outline its proof (§ 6.4.2). It might be helpful to introduce the general notion of a Hilbert scheme (§ 6.3).

Talk 12: The exact number of lines in a cubic surface

24.01., Heng Xie

Answer the following question, following §§ 5.1 and § 6.4.1: How many lines are contained in a smooth cubic surface in $\mathbb{P}^3$?