

Reading Course: Profinite Properties

Daniel Echtler

Doris Grothusmann

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Understanding infinite groups in general can be a very hard task. Looking only at finite groups should be a way easier task. Thus one might ask the question: What can one say about an infinite group by studying its finite images?

This can take multiple forms. For example the *profinite rigidity* of a group G asks, whether a group H “lying in the gutter” with the same finite quotients as G is already isomorphic to G . Another question is that of *profinite properties*, i.e. whether a property of a group can be determined by the finite quotients of a group: If G and H have the same finite quotients and G has some property (e.g. is abelian) does H also have to have the same property?

Both of these questions can be asked in an *absolute* sense, i.e. for general groups, or in a *relative* sense, i.e. for a certain class of groups, e.g. manifold groups, lattices in Lie groups, etc.

First we want to read the recent survey article *Chasing finite shadows of infinite groups through geometry* by Bridson [Bri25]. This will give us some ideas of more geometric methods used to study groups by their finite quotients. Afterwards we will look at an explicit example: The paper *Polycyclic groups, finite images, and elementary equivalence* by Sabbagh and Wilson [SW91] where it is proven that being polycyclic is a profinite property.

While reading these two papers we will occasionally have mini-talks of 10–15 minutes shedding more light onto results/concepts to understand the main content. On demand, we can also offer seminar talks for students.

In the very end of the term this program might be rounded off by a research talk on more recent results in the field.

Main references

- [Bri25] M. R. Bridson. *Chasing finite shadows of infinite groups through geometry*. preprint. 2025. arXiv: 2504.11684 [math.GR]. To appear in the Proceedings of 9ECM, the European Congress of Mathematics in Sevilla, 2024.
- [SW91] G. Sabbagh and J. S. Wilson. “Polycyclic groups, finite images, and elementary equivalence.” In: *Archiv der Mathematik* 57 (1991), pp. 221–227. DOI: 10.1007/BF01196851.

Preliminary Schedule

Date	Paper	Task
14.10.2025	[Bri25]	Read 1. INTRODUCTION
21.10.2025	[Bri25]	Read 2. PRESENTING GROUPS AND UNDERSTANDING THEM until 2.12
28.10.2025	[Bri25]	Read 3. HARD TO SEE IF FINITE IMAGES EXIST and 4. CAPTURING GROUPS VIA FINITE ACTIONS: RESIDUAL FINITENESS
04.11.2025	[Bri25]	Read 5. PROFINITE PROPERTIES OF GROUPS
11.11.2025	[Bri25]	Read 6. PROFINITE RIGIDITY
18.11.2025	[Bri25]	Read 7. GROTHENDIECK PAIRS
25.11.2025	[Bri25]	Read 8. PROFINITE RIGIDITY AND 3-MANIFOLDS and 9. ABSOLUTE PROFINITE RIGIDITY until 9.3
02.12.2025	[Bri25]	Read the remainder of Section 9., 10. SEIFERT FIBRE SPACES AND THE IMPORTANCE OF FINITENESS PROPERTIES and 11. OPEN QUESTIONS
09.12.2025	[Bri25]	Buffer
16.12.2025		Research Talk: TBD
Christmas Break		
06.01.2026	[SW91]	Read 1. INTRODUCTION, 2. PRELIMINARIES, and 3. ABELIAN GROUPS AND FINITE IMAGES
13.01.2026	[SW91]	Read 4. THE MAIN THEOREM
20.01.2026	[SW91]	Buffer or read 5. SOME LOGIC
27.01.2026		Research Talk: TBD
03.02.2026		Research Talk: TBD

Mini-Talks

Date	Paper	Task
04.11.2025	[Bri25]	What is the Nikolov–Segal Theorem quoted in the footnote on page 13?
04.11.2025	[Bri25]	What is a <i>recursively presented group</i> ? (Theorem 5.3)
18.11.2025	[Bri25]	Explain the ideas in the proof of Theorem 7.3 a bit.
25.11.2025	[Bri25]	What is a <i>foliation</i> ? (Section 8.1)
06.01.2026	[SW91]	Recall what a <i>polycyclic group</i> is.
13.01.2026	[SW91]	Provide some details for Lemma 6 and its proof (metabelian, Theorem of Hall, maximal condition for normal subgroups, ...).
