

Advanced Seminar on Group Theory – Thompson’s groups

March 2, 2021

In the 1960's Richard Thompson defined three groups nowadays denoted T , F and V that have remarkable properties, e.g.: They are finitely presented, but have unsolvable word problem, T and V are simple, F has property FP_∞ and is torsion-free. All three groups, as well as numerous generalisations, are still subject to much research.

Our plan is to follow the *Introductory Notes on Richard Thompson's Groups* [2] by Cannon, Floyd and Parry, and afterwards to look at a selection of more recent results.

TALK 1: F

Define the group F and explain what a both a rectangle and tree diagram is. Give examples! At the end, prove Theorem 2.5 in [2]. Source: §1 and §2 in [2].

TALK 2: More F

Prove that F is finitely presented. Then present some of the results of §4, preferably give a sketch of the proof of Theorem 4.5. Source: §3 and §4 in [2].

TALK 3: T

Define the group T and prove that it is simple. It is probably necessary to skip some details, but keep some example calculations to give an idea how to work with T ! Source: §5 in [2].

TALK 4: V

Define the group V , explain its relationship with T and F and define V_1 . Rather than proving that it is simple and isomorphic to V , present some explicit calculations to prove Lemma 6.1. If there is time, explain the differences and similarities between the proof of simplicity for T and V . Source: §6 in [2].

Time slots 5 and 6 should be filled with two of the following talks (**TALK X** can be chosen twice).

TALK A: F is of type FP_∞

Explain what it means to be of type FP_∞ , and follow the proof in [4] why F is of said type.

TALK B: Twisted Conjugacy Classes in F

A throwback to the topic of the summer semester 2019. Explain what twisted conjugacy classes are, and sketch the proof of Theorem 3.5 in [1].

TALK C: Cloning Systems

This is a far-reaching generalisation of Thompson's groups in terms of a limiting process, introduced by Zaremsky and Witzel. Follow along the *User's Guide to Cloning Systems* [3], that *distill[s] the crucial parts of that 50-page paper into a more digestible form, for those curious to understand the construction but less curious about the gritty details*. Concentrate on your favourites among the many examples.

TALK X: Your favourite result about Thompson's groups

Show use whatever pleases you.

References

- [1] Bleak, Collin; Fel'shtyn, Alexander; Gonçalves, Daciberg L. *Twisted conjugacy classes in R. Thompson's group F*. Pac. J. Math. 238, No. 1, 1-6 (2008).
- [2] Cannon, J. W.; Floyd, W. J.; Parry, W. R. *Introductory notes on Richard Thompson's groups*. Enseign. Math., II. Sér. 42, No. 3-4, 215-256 (1996).
- [3] Zaremsky, Matthew C. B. *A user's guide to cloning systems*. Topol. Proc. 52, 13-33 (2018).
- [4] Zaremsky, Matthew C. B. *A short account of why Thompson's group F is of type F_∞* . Topol. Proc. 57, 77-86 (2021).