In this seminar, we will explore groups acting on regular rooted trees. As we will see, these groups provide a rich source of examples with interesting properties in group theory and have been instrumental in solving important mathematical problems.

For instance, the first Grigorchuk group, introduced by Rostislav Grigorchuk in 1980, is one of the first known examples of an infinite, finitely generated periodic group, thereby providing a negative solution to the General Burnside Problem. It is also the first example of a group with intermediate growth, resolving Milnor's Problem. Additionally, the Grigorchuk group is amenable but not elementary amenable, has a solvable word problem, is commensurable with its own direct product, is just-infinite, and possesses many other remarkable properties.

The aim of this seminar is to read and understand the papers "Dimension and Randomness in Groups Acting on Rooted Trees" by Miklós Abért and Bálint Virág and "Amenability via Random Walks" by Laurent Bartholdi and Bálint Virág. To achieve this, we will begin by introducing groups acting on regular rooted trees and then develop some of the core ideas that will later be required to understand the aforementioned papers.

Seminar Structure

Introduction (1–2 Seminars) See [diD22], [Gri00], [Nek05], [dlH00].

The first one or two seminars will be dedicated to introducing fundamental concepts related to groups acting on regular rooted trees. Topics will include the definition of automorphisms, the full automorphism group Aut(T), stabilizers, self-similar groups, branch groups, and more—all while keeping the Grigorchuk group as our motivating example.

TBC: Development of Core Ideas (3–5 Seminars)

- **Seminar 3** See [diD22], [FAZR14], **:** We will introduce some essential tools for working with groups acting on regular rooted trees. In particular, we will present criteria for determining whether a group is self-similar, fractal, level-transitive, (weakly) branch, (weakly) regular branch, or belongs to the class MF (groups in which all maximal subgroups have finite index).
- **Seminar 4:** We will examine the structure of Aut(T) as a profinite group. This perspective allows us to study the topological properties of groups acting on regular rooted trees, such as the Hausdorff dimension and the congruence subgroup property. We will also discuss how this relates to the concept of fractality.

- **Seminar 5:** We will discuss a special subclass of self-similar groups called automata groups, which have useful algorithmic properties. We will also explore how these groups relate to groups acting on regular rooted trees. In particular, we will see that the action of the Grigorchuk group on the regular rooted tree can also be described as a finite-state automaton.

TBC: Reading Seminars

- **(Reading) seminars 6–9:** We will read and analyze the first half of the paper *"Dimension and Randomness in Groups Acting on Rooted Trees"* by Miklós Abért and Bálint Virág.

- **(Reading) seminars 10—13:** We will read and study *"Amenability via Random Walks"* by Laurent Bartholdi and Bálint Virág.

Flexibility and Adjustments

Depending on the interests of the seminar participants, we may adjust the topics covered to better align with the group's preferences. Also some topics might take either more or less time than expected.

Literature

Introduction and Development of Core Ideas:

[diD22] E. Di Domenico. Some questions regarding groups of automorphisms of primary trees. Ph.D. thesis.
Very recommended. Download link: https://addi.ehu.eus/bitstream/handle/10810/56168/TESIS_ELENA_DI_DOMENICO.pdf?sequence=1&isAllowed=y

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[Nek05] V. Nekrashevych. Self-similar groups, volume 117 of Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2005.

[dlH00] P. de la Harpe. Topics in geometric group theory. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 2000.

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[UA16] J. Uria-Albizuri, On the concept of fractality for groups of automorphisms of a regular rootedtree, Reports@SCM 2 (2016), 33–44.

[Fal90] K. Falconer. Fractal geometry-mathematical foundations and applications. Wiley, 1990.

[Abe94] A. G. Abercrombie, Subgroups and subrings of profinite rings, Math. Proc. Camb. Phil. Soc. 116 (2) (1994), 209–222.

[BS97] Y. Barnea and A. Shalev, Hausdorff dimension, pro-p groups, and Kac-Moody algebras, Trans.Amer. Math. Soc. 349 (1997)

Reading Seminars:

[AV22] M. Abert and B. Virág. Dimension and randomness in groups acting on rooted trees, volume 18, number 1 of Journal of the American society, pages 157–192, 2004.

[BV05] L. Bartholdi and B. Virág. Amenability via Random Walks. Duke Math. J. 130 (1) 39 - 56, 01 2005.