Advanced Seminar on Group Theory

Right-angled Artin groups

February-March 2022

The goal of this seminar is to give a gentle introduction to the theory of right-angled Artin groups, RAAGs for short. These groups span a wide range of groups from finitely generated free groups to finitely generated free abelian groups, and posses a rich structure of subgroups as well as nice algorithmic properties. The seminar will consist of five talks, each of them devoted to different topics exposed in the survey papers [4] and [11]; or in the book [8, Office hour 14].

Talk 1: 23.02.2022 — **The word and conjugacy problems in RAAGs.** Give a general and brief introduction to the theory of RAAGs (following, for example, [11]). Show that there exist linear-time algorithms for the word and conjugacy problems in RAAGs (as proved in [5]). In particular, introduce the notion of pilings and briefly explain how these can be used to extend the result to an infinite family of subgroups of RAAGs.

Talk 2: 02.03.2022 — **Centralizers of RAAGs.** Prove that centralizers of elements of RAAGs are themselves RAAGs. Read [8, Section 2.1] for a nice description of the result and follow [12, Section 3] for its proof (this result was originally proved in [1] by Baudisch, but the paper is in German and the notation is much more obscure). Devote also some time to talk about the result in [2] concerning 2-generator subgroups of RAAGs.

Talk 3: 09.03.2022 — **Special subgroups of RAAGs.** Following [9], give sufficient and necessary conditions for every subgroup of RAAGs to be RAAGs. Talk also about the characterisation given in [10] of the RAAGs all of whose subgroups are coherent. A nice overview of the topic can be found in [8, Section 3].

Talk 4: 14.03.2022 — **Coxeter groups and the linearity of RAAGs.** Introduce the notion of Coxeter group and give some basic examples (you can find them, for instance, in [3, Chapter IV, Section 1.3]). Define Coxeter matrices and Coxeter graphs (see [3, Chapter IV, Section 1.9]), and, with these notions, sketch the proof in [3, Chapter V, Section 4.4, Corollary 2] for the linearity of Coxeter groups.

RAAGs can be realized as quotients of Coxeter groups. Following [7], show that RAAGs actually embed as finite index subgroups of Coxeter groups. This, in particular, shows that RAAGs are linear.

Talk 5: 21.03.2022 — **Tits' conjecture.** It was conjectured by Tits that the subgroup generated by the squares of the generators of an arbitrary Artin group is always a RAAG. This was shown to be true by Crisp and Paris in 2001 [6]. Sketch the proof for this result omitting, if necessary, the details in Sections 4 and 5.

References

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