Singularities, Monodromy and Zeta functions
Blatt 1

Exercises for discussion in Übung on 18.10.2018

Not all of the following exercises have been fully tested. If any seem to be impossible, they might be! You are welcome to ask if you are stuck.

A possible list of polynomials $f \in \mathbb{Z}[x_1, ..., x_n]$ you might choose as examples for Aufgabe 1 - 3 are:

(a) $f_1(x) = x^k$;

(b) $f_2(x, y) = xy$;

(c) For $k, m \in \mathbb{N}$, $f_3(x, y) = x^k y^m$;

(d) $f_4(x) = x^2 + x$;

(e) $f_5(x, y, z) = x^2 + y^2 + z^2$;

(f) $f_6(x, y) = x^3 - y^2$;

(g) For $k, m \in \mathbb{N}$, $f_7(x, y) = x^k - y^m$.

You are welcome to choose other polynomials!

Aufgabe 1:

Compute the Poincaré series $P_p(T)$ associated to some choices of polynomials $f \in \mathbb{Z}[x_1, ..., x_n]$.

Aufgabe 2:

Consider the ring of differential operators $D_n = \mathbb{Z}[x_1, ..., x_n, \partial_{x_1}, ..., \partial_{x_n}]$.

It has been claimed that, given $f \in \mathbb{Z}[x_1, ..., x_n]$, there are $P \in D_n$ and some polynomial $b(s) \in \mathbb{Q}[s]$ such that,

$$P(f^{s+1}) = b(s) f^s.$$

(a) For some choices of polynomials f, find such an operator P and polynomial $b(s)$ satisfying the above relationship.

(b) After you have found some examples, do you have any conjectures about P?

Aufgabe 3:

Compute the Igusa p-adic (local) Zeta function $Z_p(s)$ associated to some choices of polynomials $f \in \mathbb{Z}[x_1, ..., x_n]$.

Aufgabe 4:

Suppose $T = p^{-s}$ for $\mathrm{Re}(s) > 0$. Prove or maybe correct the following expression of a relationship between the Poincaré series P_p and the Igusa p-adic (local) Zeta function Z_p:

$$P_p(Tp^{-n}) = \frac{Z_p(s)T + 1}{T - 1}.$$