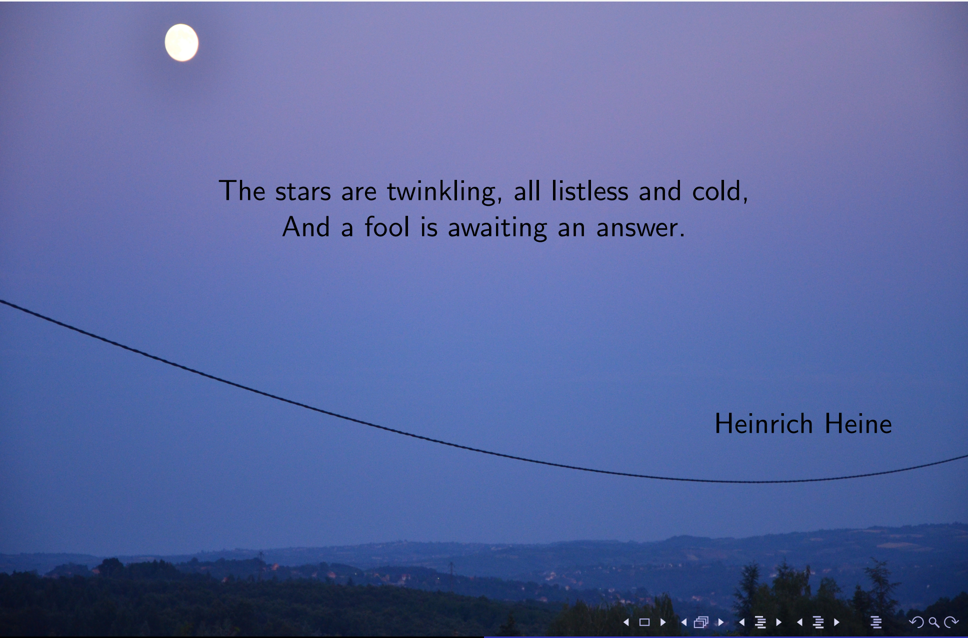
from "Questions"



The stars are twinkling, all listless and cold,
And a fool is awaiting an answer.

Heinrich Heine

Group tree shifts III

Zoran Šunić
Hofstra University

* * *

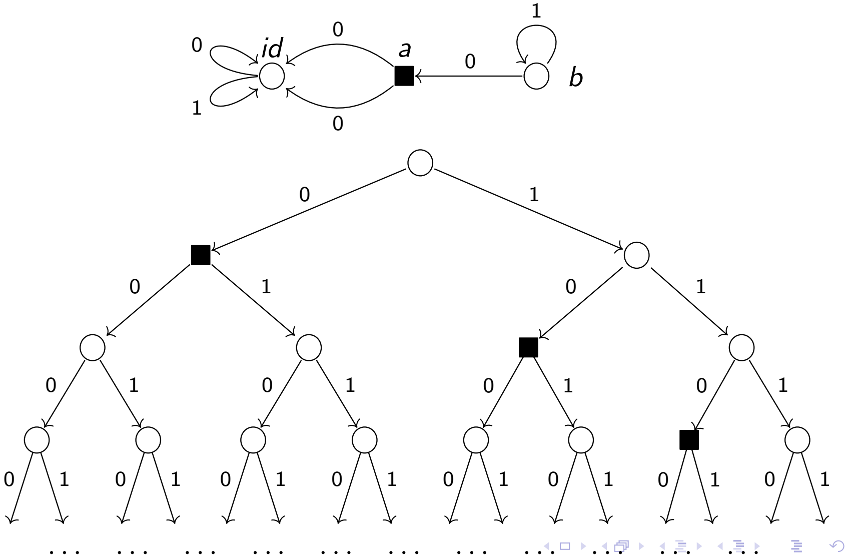
Düsseldorf (Heinrich-Heine-Universität), June 28, 2018

Statue of Lorelei on the occasion of Heine's 100th ...

Moore automaton (defines finite self-similar sets)

a finite graph S with
- vertices labeled by permutations in Σ and,
- for each state $s \in S$, one outgoing bundle of edges which are labeled bijectively by X.
- a portrait is accepted if there is a vertex and edge label preserving homomorphism from the portrait to the automaton (exactly one portrait per state is accepted/recognized/defined)

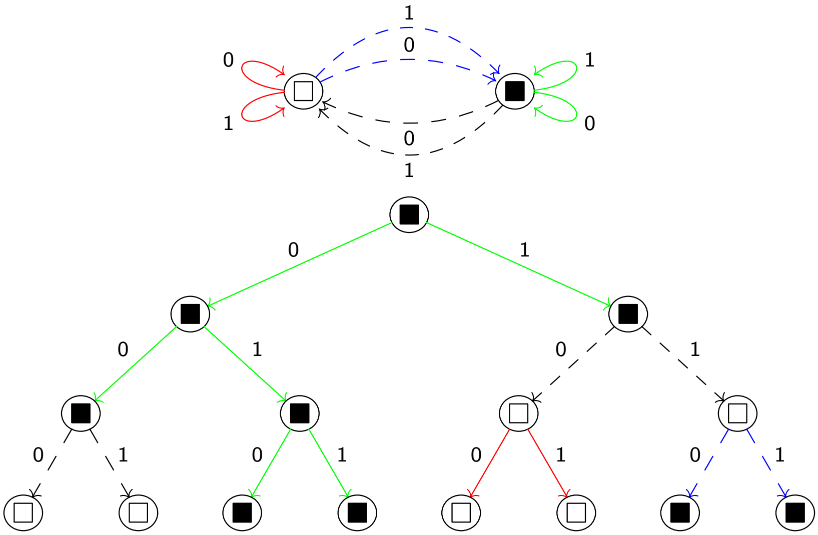
Moore automaton (defines finite self-similar sets)



Rabin-Moore automaton (defines closed self-similar sets)

a finite graph S with
- vertices labeled by permutations in Σ and,
- for each state $s \in S$, several outgoing bundles of edges, with the edges in each bundle labeled bijectively by X.
- a portrait is accepted if there is a vertex and edge label preserving and bundle preserving homomorphism from the portrait to the automaton

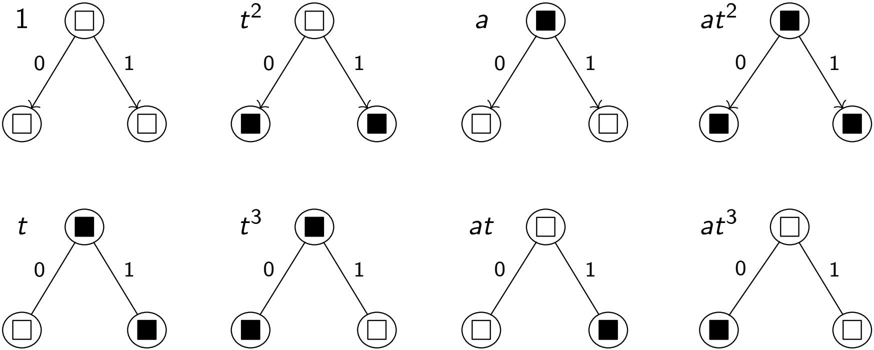
Rabin-Moore automaton (defines closed self-similar sets)



What exactly is the set (group) recognized?

$$\mathcal{G}_{\mathcal{B}} = \{\ g \in \Sigma^{X^*} \mid (\forall u \in X^*) g|_{(u0)} = g|_{(u1)}\ \}$$

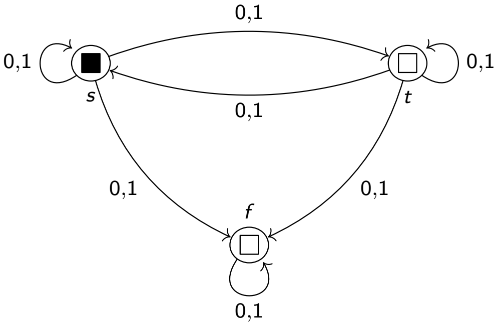
(Group) tree shift of finite type, defined by forbidden tree-patterns of size 2 (the patterns in the bottom row are forbidden).



Rabin automaton (S2S definable sets)

a finite graph S with
- vertices labeled by permutations in Σ and,
- for each state $s \in S$, several outgoing bundles of edges, with the edges in each bundle labeled bijectively by X.
- set of initial states I
- family F of sets of states (final sets of states)
- a portrait is accepted if (1) there is a vertex and edge label preserving and bundle preserving homomorphism from the portrait to the automaton and (2) for each ray, the set of states visited by the ray under the accepting homomorphism is a member of F

Rabin automaton (S2S definable sets)



$$I = \{s, t, f\}$$

$$F = \{\{f\}\}$$

S2S: Monadic second order theory of the two successors structure X^*

first order varaiables: $x, y, z, \ldots$ (range over vertices in X^*)
second order varaibles: $X, Y, Z, \ldots$ (range over subsets of X^*)
2 successors: s_0 and s_1 ($s_0(u)$ and $s_1(u)$ interpreted as the successors of u in X^*)

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equality symbol and membership symbol: $=$, $\in$
quantifiers: $\forall, \exists$
logical connectives: $\neg, \wedge, \vee, \implies$
parentheses: (and)
basic terms: x
terms: for every term t, the expressions $s_0(t)$ and $s_1(t)$ are terms.
atomic formulas: $t_1 = t_2$, $X = Y$, $t \in X$.
formulas: for all formulas Φ and Ψ the following are also formulas

$$(\neg\Phi),\ (\Phi \wedge \Psi),\ (\Phi \vee \Psi),\ (\Phi \implies \Psi),$$
$$(\exists x\ \Phi),\ (\exists X\ \Phi),\ (\forall x\ \Phi),\ (\forall X\ \Phi).$$

Rabin Theorem(s)

Theorem (Rabin 1969)

The S2S theory of the two successor structure X^* is decidable.

Theorem (Rabin 1969)

A set of labeled trees is definable in S2S if and only if it can be recognized by a Rabin automaton.

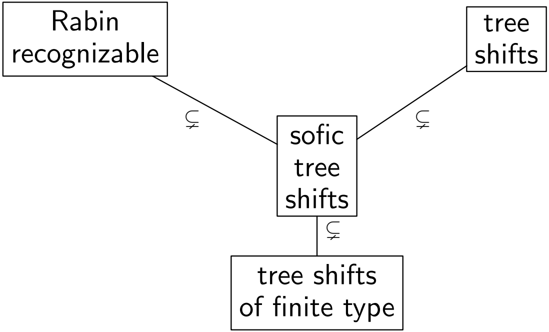
Still one of the strongest decidability results in logic (used to prove decidability of many other theories by embedding them in S2S)

Sofic shifts and shifts of finite type (C-S,C,F,Š, 2013)

defined by frbddn ptrns	$\overset{thm}{\Longleftrightarrow}$	Subshift (closed and shift-invariant set)		
		$\supsetneq$		
quotient of a SFT	$\overset{thm}{\Longleftrightarrow}$	Sofic shift	$\overset{def}{\Longleftrightarrow}$	recognizable by a Rabin autom. no restrictions (Rabin-Moore)
		$\supsetneq$		
defined by finitely many frbddn pttrns	$\overset{def}{\Longleftrightarrow}$	Shift of finite type	$\overset{thm}{\Longrightarrow}$	recognizable by a Rabin autom. no restrictions (Rabin-Moore)

Sofic = Rabin rcgnzble ∩ shifts = S2S dfnble ∩ shifts

Ceccherini-Silberstein, Coornaert, Fiorenzi, Š, 2013



The shift-invariance (self-similarity) removes the need for initial states and the topological closure property removes the need for final states.

A simple observation

Proposition

Every group tree shift of finite type (in fact, every sofic group tree shift) is a closure of a group of finite-state automorphisms.

Proof.

Let G be a sofic group tree shift accepted by the Rabin-Moore automaton $\mathcal{A}$. For each state of $\mathcal{A}$ fix an outgoing bundle (thus, select a Moore automaton $\mathcal{A}'$ inside the Rabin-Moore automaton $\mathcal{A}$). Any $g \in G$ can be approximated up to a given level by a finite-state automorphism g' in G that "imitates" g in $\mathcal{A}$ up to that level and follows $\mathcal{A}'$ below that level. □

A characterization of group tree shifts of finite type

Theorem (Grigorchuk 2006 (i) implies (ii), Š. 2007 converse)

Let G be a subgroup of $\mathrm{Aut}(X^*)$. The following are equivalent.
(i) *G is defined by forbidden tree patterns of size s*
(ii) *G is the closure of a regular branch group H, branching over its stabilizer H_{s-1} of level $s-1$.*

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- ($\subseteq$) This is always true in a self-similar group.
If h stabilizes s levels then its sections $h_0, h_1, \dots, h_{k-1}$ must stabilize $s-1$ levels.

- ($\supseteq$) If $h_0, h_1, \dots, h_{k-1}$ stabilize $s-1$ levels then $h = (h_0, h_1, \dots, h_{k-1})$ stabilizes s levels.

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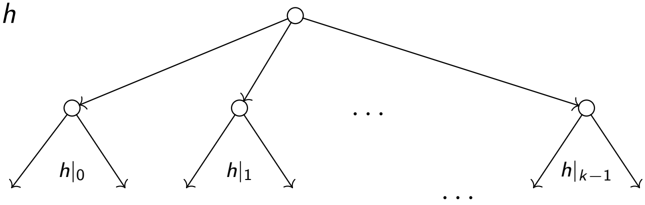
Yes, but is it in H?

Group H branching over the stabilizer of level $s-1$

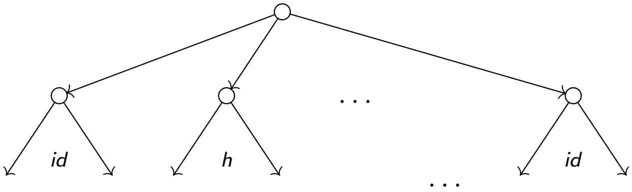
$$H_s = H_{s-1} \times H_{s-1} \times \cdots \times H_{s-1}$$



Group H branching over the stabilizer of level $s-1$

$$H_s = H_{s-1} \times H_{s-1} \times \cdots \times H_{s-1}$$

Thus, the real question is:
given h in H_{s-1}, is the following automorphism in H (for every position on the first level, of course)?



self-similar = every section is an element

= anything can be brought from the anywhere to the root

$= (\forall h \in H)(\forall u \in X^*)\ h|_u \in H$

self-replicating = every element is a (special) section

= anything can be taken from the root to anywhere

$= (\forall h \in H)(\forall u \in X^*)(\exists g \in \mathrm{Stab}_H(u))\ g|_u = h$

branch over H_{s-1} = every element in H_{s-1} is a (very special) section

= anything in H_{s-1} can be taken from the root to anyw

$= (\forall h \in H)(\forall u \in X^*)(\exists g \in \mathrm{Rist}_H(u))\ g|_u = h$

Two examples

Example

The first Grigorchuk group (1980) branches over its stabilizer of level 3, so its closure is a group tree shift of finite type defined by patterns of size 4
(total of $2^{12} = 4096$ allowed patterns; thus $2^{15} - 2^{12}$ forbidden).

Example

The Gupta-Sidki 3-group (1983) branches over its stabilizer of level 2, so its closure is a group tree shift of finite type defined by patterns of size 3
(total of $3^8 = 6561$ allowed patterns; thus $3^{13} - 3^8$ forbidden).

Third example

Theorem (Grigorchuk 2006 (i) implies (ii), Š. 2007 converse)

Let G be a subgroup of $\mathrm{Aut}(X^*)$. The following are equivalent.
(i) *G is defined by forbidden tree patterns of size s*
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Example (Warning)

The closure $\overline{H}$ of the Hanoi Towers group H (on 3 pegs) is a finitely constrained grup defined by ptterns of size 2, even though H does not branch over a level stabilizer.

Third example

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Example (Warning)

The closure $\overline{H}$ of the Hanoi Towers group H (on 3 pegs) is a finitely constrained grup defined by ptterns of size 2, even though H does not branch over a level stabilizer.
The point is that the closure itself does branch over a level stabilizer.

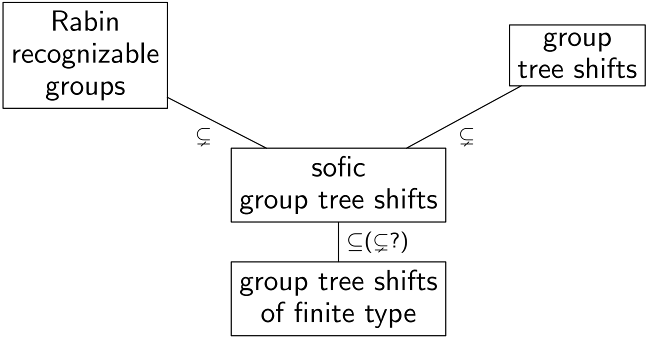
Is sofic actually different from finite type?

I do not know any examples of a sofic group tree shift that is not group tree shift of finite type.

Theorem (Penland, Š 2018)

Let G be a sofic group tree shift such that its normalizer in $\mathrm{Aut}(X^*)$ *contains a level transitive self-replicating subgroup. Then G is a group tree shift of finite type.*

Sofic = Rabin rcgnzble ∩ shifts = S2S dfnble ∩ shifts



$\mathbb{Z}_2$ is not S2S definable

Corollary (Penland, Š 2018)

Let G be a level transitive, self-replicating group that satisfies a group identity (or has a nontrivial center or ...). Then, the closure $\overline{G}$ is a group tree shift that is not Rabin recognizable (and, in particular, it is not sofic or of finite type).

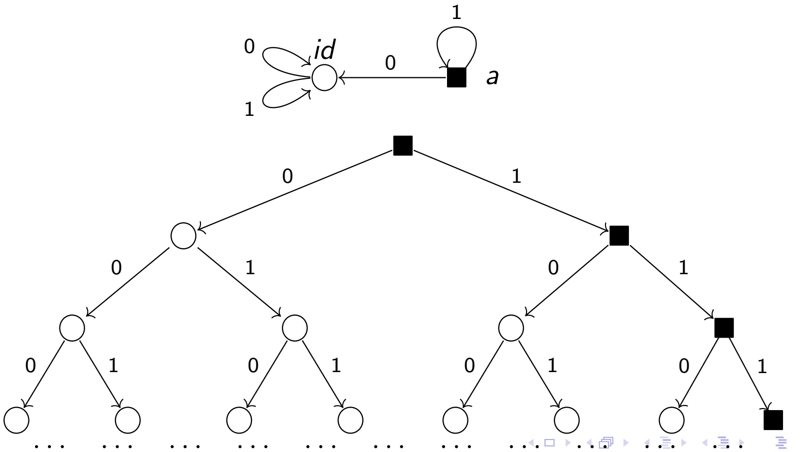
Proof.

If $\overline{G}$ were Rabin recognizable, it would be sofic, but then it would be of finite type, but then it would be branch, but then it could not satisfy an identity (note that $\overline{G}$ satisfies all identities that G does), a contradiction. □

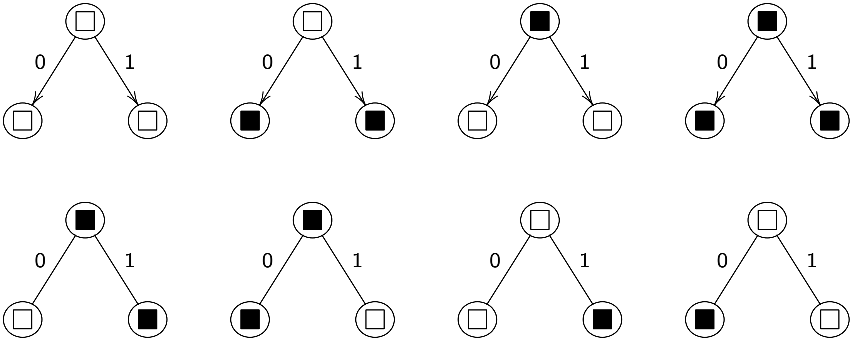
Adding machine

That foul machine! and God forbid
That I should ever use it!

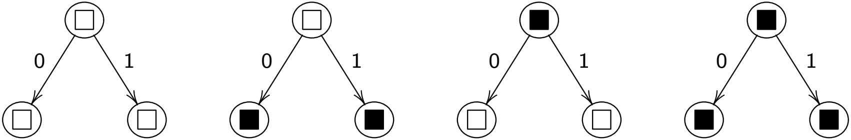
Heinrich Heine



Tree patterns (of size 2)

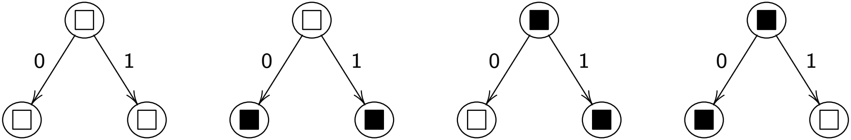


Two groups defined by patterns



These patterns define the "same children" group

$$G_{sc} = \{ \; g \in \Sigma^{X^*} \mid g_{(u0)} = g_{(u1)}, \text{ for all } u \; \}$$



These patterns define the "even pattern" group

$$G_{even} = \{ \; g \in \Sigma^{X^*} \mid g_{(u0)} + g_{(u1)} + g_{(u)} = 0, \text{ for all } u \; \}$$

Finitely constrained groups

Definition

A group of tree automorphism defined by a finite set of forbidden patterns is called a **finitely constrained group** (group tree shift of finite type)

The patterns for the Grigorchuk group have **3** constraints

- Grigorchuk 2000 (indirectly)
- Arzhantseva-ZŠ 2007 (3 explicitly stated constraints)
- Bartholdi around 2007 (explicitly stated "single" constraint)

"degrees of freedom" $= 5$ out of 8.

Entropy of f.c.g. with respect to balls on the tree

$$\lim_{n\to\infty} \frac{\log_2(\# \text{ of patterns of size } n \text{ appearing in } G)}{\log_2(\# \text{ of all possible patterns of size } n)} =$$

$$\frac{(1+2+\cdots+2^{n-1}) - 3(1+2+\cdots+2^{n-4})}{1+2+\cdots+2^{n-1}} =$$

$$1 - \frac{3}{8} = \frac{5}{8}$$

$$\mathrm{ent}_B(G) = \lim_{n\to\infty} \frac{\log_2(\# \text{ of patterns on the ball of radius } n-1)}{\text{size of the ball of radius } n-1}$$

Hausdorff dimension of closed subgroups of $\mathrm{Aut}(X^*)$

Theorem (Abercrombie 1994, Barnea-Shalev 1997)

$$\dim_H H = \lim_{n\to\infty} \frac{\log[G : G_n]}{\log[\mathrm{Aut} : \mathrm{Aut}_n]}$$

Two portraits are "close" when they agree on "many" levels:

$$d(g,h) = \inf\{ \frac{1}{[\mathrm{Aut} : \mathrm{Aut}_n]} \mid (\forall m \leq n)(\forall u \in X^m)\ g|_{(u)} = h|_{(u)} \}$$

Thus, for finitely constrained groups on the binary tree

$$\mathrm{ent}_B(G) = \dim_H(G).$$

(For other trees we have $\mathrm{ent}_B(G) = \dim_H(G) \log_2 |\Sigma|$.)

From now on – binary trees

Look mom – no limits (Š, Bartholdi, Bondarenko-Samoilovych, ...)

For a finitely constrained group defined by patterns of size s, for some $s \geq 1$,

$$\dim_H(G) \in \left\{1,\ 1 - \frac{1}{2^{s-1}},\ 1 - \frac{2}{2^{s-1}},\ 1 - \frac{3}{2^{s-1}}, \dots, \frac{1}{2^{s-1}},\ 0\right\}$$

$$\dim_H(G) = \frac{\log_2 |P_{s-1}|}{2^{s-1}}$$

where P is the essential finite group of patterns of size s defining G.

essential group of patterns = the group of allowed patterns that actually appear in some element

	1	$1-\frac{1}{2^{s-1}}$		$1-\frac{2}{2^{s-1}}$		$1-\frac{3}{2^{s-1}}$	
$s=1$	$\mathrm{Aut}(X^*)$	0	trivial	X	XXXXX	X	XXXXX
$s=2$	$\mathrm{Aut}(X^*)$	$\frac{1}{2}$		0	finite	X	XXXXX
$s=3$	$\mathrm{Aut}(X^*)$	$\frac{3}{4}$		$\frac{2}{4}$		$\frac{1}{4}$	
$s=4$	$\mathrm{Aut}(X^*)$	$\frac{7}{8}$		$\frac{6}{8}$		$\frac{5}{8}$	RG "2005"
$s=5$	$\mathrm{Aut}(X^*)$	$\frac{15}{16}$		$\frac{14}{16}$		$\frac{13}{16}$	
$s=6$	$\mathrm{Aut}(X^*)$	$\frac{31}{32}$		$\frac{30}{32}$		$\frac{29}{32}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s	$\mathrm{Aut}(X^*)$						

Is there a top.fin.gen. finitely constrained group of a given dimension defined by patterns of a given size?

BLUE = YES
RED = NO
BLACK = the question is trivial
XXXXX = question makes no sense (negative dimension)

Iterated monodromy groups of the tent map

Theorem (ZŠ 2007)

For every $s \geq 4$, there are at least 2^{s-2} topologically finitely generated, finitely constrained groups defined by patterns of size s (but not by patterns of smaller size) whose Hausdorff dimension is

$$1 - \frac{3}{2^{s-1}}$$

The groups above are "parametrized" by polynomials over GF(2). The two groups corresponding to size 4 are the Grigorchuk group is G_{x^2+x+1} and Grigorchuk-Erschler group is G_{x^2+1}.

Theorem (Nekrashevych-ZŠ)

The groups above together with the dihedral group correspond to the iterated monodromy groups of the self-cover of the unit interval by the tent map.

So far (ZŠ 2007)

	1		$1-\frac{1}{2^{s-1}}$		$1-\frac{2}{2^{s-1}}$		$1-\frac{3}{2^{s-1}}$
$s=1$	$\mathrm{Aut}(X^*)$	0	trivial	X	XXXXX	X	XXXXX
$s=2$	$\mathrm{Aut}(X^*)$	$\frac{1}{2}$		0	finite	X	XXXXX
$s=3$	$\mathrm{Aut}(X^*)$	$\frac{3}{4}$		$\frac{2}{4}$		$\frac{1}{4}$	
$s=4$	$\mathrm{Aut}(X^*)$	$\frac{7}{8}$		$\frac{6}{8}$		$\frac{5}{8}$	RG 2005
$s=5$	$\mathrm{Aut}(X^*)$	$\frac{15}{16}$		$\frac{14}{16}$		$\frac{13}{16}$	ZŠ 2007
$s=6$	$\mathrm{Aut}(X^*)$	$\frac{31}{32}$		$\frac{30}{32}$		$\frac{29}{32}$	ZŠ 2007
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s	$\mathrm{Aut}(X^*)$						ZŠ 2007

Is there a top.fin.gen. finitely constrained group of a given dimension defined by patterns of a given size?

BLUE = YES
RED = NO

The two f.c groups defined by patterns of size 2

$$\dim_H(\text{even tree shift group}) = \dim_H(\text{same children group}) = \frac{1}{2}$$

Both are defined by a single constraint and have maximal Hausdorff dimension for the given size.

Theorem (ZŠ 2011)

Neither of these two groups is topologically finitely generated.

So far (ZŠ 2011)

	1	$1-\frac{1}{2^{s-1}}$		$1-\frac{2}{2^{s-1}}$		$1-\frac{3}{2^{s-1}}$	
$s=1$	$\mathrm{Aut}(X^*)$	0	trivial	X	XXXXX	X	XXXXX
$s=2$	$\mathrm{Aut}(X^*)$	$\frac{1}{2}$	ZŠ 2011	0	finite	X	XXXXX
$s=3$	$\mathrm{Aut}(X^*)$	$\frac{3}{4}$		$\frac{2}{4}$		$\frac{1}{4}$	
$s=4$	$\mathrm{Aut}(X^*)$	$\frac{7}{8}$		$\frac{6}{8}$		$\frac{5}{8}$	RG "2005"
$s=5$	$\mathrm{Aut}(X^*)$	$\frac{15}{16}$		$\frac{14}{16}$		$\frac{13}{16}$	ZŠ 2007
$s=6$	$\mathrm{Aut}(X^*)$	$\frac{31}{32}$		$\frac{30}{32}$		$\frac{29}{32}$	ZŠ 2007
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s	$\mathrm{Aut}(X^*)$						ZŠ 2007

Is there a top.fin.gen. finitely constrained group of a given dimension defined by patterns of a given size?

BLUE = YES
RED = NO

F.c. groups defined by patterns of size 3 and size 4

Theorem (Bondarenko-Samoilovych 2013)

No finitely constrained group defined by a pattern group P of pattern size 3 is topologically finitely generated.

Theorem (Bondarenko-Samoilovych 2013)

No finitely constrained group defined by a pattern group P of pattern size 4 and Hausdorff dimension different from $5/8$ is topologically finitely generated.

So far (IB-IS 2013)

	1	$1 - \frac{1}{2^{s-1}}$		$1 - \frac{2}{2^{s-1}}$		$1 - \frac{3}{2^{s-1}}$	
$s = 1$	$\mathrm{Aut}(X^*)$	0	trivial	X	XXXXX	X	XXXXX
$s = 2$	$\mathrm{Aut}(X^*)$	$\frac{1}{2}$	ZŠ 2011	0	finite	X	XXXXX
$s = 3$	$\mathrm{Aut}(X^*)$	$\frac{3}{4}$	IB-IS 2013	$\frac{2}{4}$	IB-IS 2013	$\frac{1}{4}$	IB-IS 2013
$s = 4$	$\mathrm{Aut}(X^*)$	$\frac{7}{8}$	IB-IS 2013	$\frac{6}{8}$	IB-IS 2013	$\frac{5}{8}$	RG "2005"
$s = 5$	$\mathrm{Aut}(X^*)$	$\frac{15}{16}$		$\frac{14}{16}$		$\frac{13}{16}$	ZŠ 2007
$s = 6$	$\mathrm{Aut}(X^*)$	$\frac{31}{32}$		$\frac{30}{32}$		$\frac{29}{32}$	ZŠ 2007
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
s	$\mathrm{Aut}(X^*)$						ZŠ 2007

Is there a top.fin.gen. finitely constrained group of a given dimension defined by patterns of a given size?

BLUE = YES
RED = NO

A sufficient condition

Theorem (Bondarenko-Samoilovych 2013)

If G is a finitely constrained group defined by a pattern group P of pattern size s and $[P, P]$ does not contain $\mathrm{Stab}_P(s-1)$ then G is not topologically finitely generated.

In particular, abelian pattern groups lead to groups that are not topologically finitely generated.

Theorem (Penland-ZŠ 2015)

If G is a finitely constrained group defined by a pattern group P of pattern size s, for some $s \geq 2$, and G has maximal Hausodrff dimension (equal to $1 - \frac{1}{2^{s-1}}$), then
(a) P is a maximal subgroup of $\mathrm{Aut}(X^{[s]}) = \underbrace{C_2 \wr C_2 \wr \cdots \wr C_2}_{s}$ that does not contain the "last generator" and
(b) G is not topologically finitely generated.

So far (AP-ZŠ 2015)

	1		$1-\frac{1}{2^{s-1}}$		$1-\frac{2}{2^{s-1}}$		$1-\frac{3}{2^{s-1}}$
$s=1$	$\mathrm{Aut}(X^*)$	0	trivial	X	XXXXX	X	XXXXX
$s=2$	$\mathrm{Aut}(X^*)$	$\frac{1}{2}$	ZŠ 2011	0	finite	X	XXXXX
$s=3$	$\mathrm{Aut}(X^*)$	$\frac{3}{4}$	IB-IS 2013	$\frac{2}{4}$	IB-IS 2013	$\frac{1}{4}$	IB-IS 2013
$s=4$	$\mathrm{Aut}(X^*)$	$\frac{7}{8}$	IB-IS 2013	$\frac{6}{8}$	IB-IS 2013	$\frac{5}{8}$	RG "2005"
$s=5$	$\mathrm{Aut}(X^*)$	$\frac{15}{16}$	AP-ZŠ 2015	$\frac{14}{16}$		$\frac{13}{16}$	ZŠ 2007
$s=6$	$\mathrm{Aut}(X^*)$	$\frac{31}{32}$	AP-ZŠ 2015	$\frac{30}{32}$		$\frac{29}{32}$	ZŠ 2007
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
s	$\mathrm{Aut}(X^*)$		AP-ZŠ 2015				ZŠ 2007

Is there a top.fin.gen. finitely constrained group of a given dimension defined by patterns of a given size?

BLUE = YES
RED = NO

It follows from the work of Bartholdi-Nekrashevych, Pink, (Penland-Š) that

Proposition

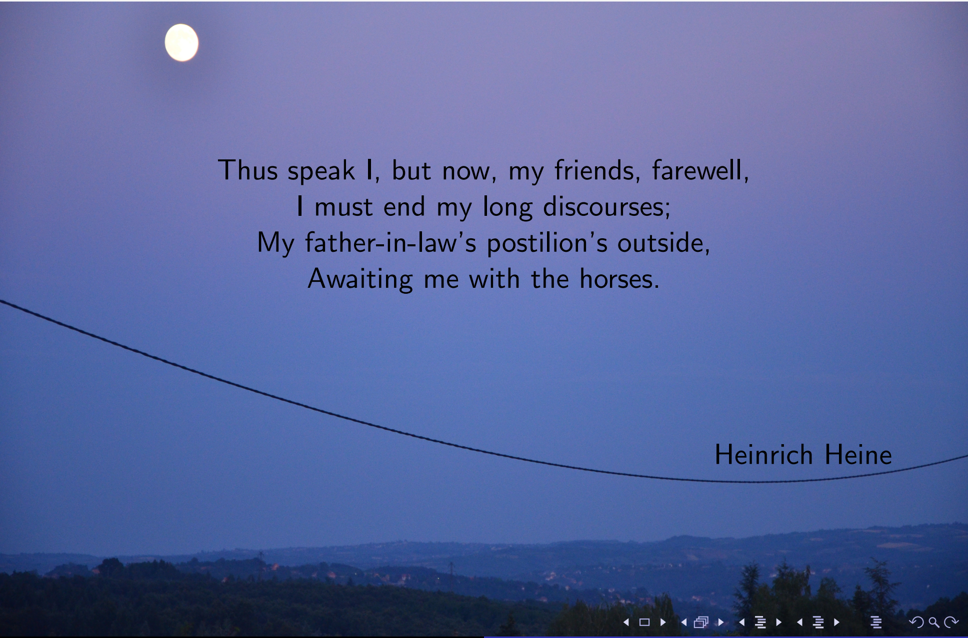
The iterated monodromy groups of post-critically finite quadratic polynomials yield finitely constrained, topologically finitely generated groups defined by patterns of size s and dimension $1 - 2/2^{s-1}$, for $s \geq 5$.

So far (B-N,P 2015)

	1	$1-\frac{1}{2^{s-1}}$		$1-\frac{2}{2^{s-1}}$		$1-\frac{3}{2^{s-1}}$	
$s=1$	$\mathrm{Aut}(X^*)$	0	trivial	X	XXXXX	X	XXXXX
$s=2$	$\mathrm{Aut}(X^*)$	$\frac{1}{2}$	ZŠ 2011	0	finite	X	XXXXX
$s=3$	$\mathrm{Aut}(X^*)$	$\frac{3}{4}$	IB-IS 2013	$\frac{2}{4}$	IB-IS 2013	$\frac{1}{4}$	IB-IS 2013
$s=4$	$\mathrm{Aut}(X^*)$	$\frac{7}{8}$	IB-IS 2013	$\frac{6}{8}$	IB-IS 2013	$\frac{5}{8}$	RG "2005"
$s=5$	$\mathrm{Aut}(X^*)$	$\frac{15}{16}$	AP-ZŠ 2015	$\frac{14}{16}$	B-N,P 2015	$\frac{13}{16}$	ZŠ 2007
$s=6$	$\mathrm{Aut}(X^*)$	$\frac{31}{32}$	AP-ZŠ 2015	$\frac{30}{32}$	B-N,P 2015	$\frac{29}{32}$	ZŠ 2007
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
s	$\mathrm{Aut}(X^*)$		AP-ZŠ 2015		B-N,P 2015		ZŠ 2007

Is there a top.fin.gen. finitely constrained group of a given dimension defined by patterns of a given size?

BLUE = YES
RED = NO



Thus speak I, but now, my friends, farewell,
I must end my long discourses;
My father-in-law's postilion's outside,
Awaiting me with the horses.

Heinrich Heine