# Mini-courses

## Topological dynamics and groups

VOLODYMYR NEKRASHEVYCH  Texas A&M University

Very often interesting groups are defined by their action on a topological space. In many cases the action itself is easier to describe and to study than the properties of the group. Nevertheless, there are many techniques allowing to draw conclusions about the structure of a group from the dynamical properties of its action. We will discuss several instances of this approach, mostly related to finiteness properties: amenability, torsion, sub-exponential growth. We will see how dynamical systems become a rich source of examples of groups with unusual properties and how group theory can be used in dynamics and vice versa.

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## Group tree shifts

ZORAN ŠUNIĆ  Hofstra University

The set of automorphisms of a rooted tree can be considered in several settings: group theoretic, topological, or symbolic dynamics. In a series of three lectures, we survey group tree shifts, that is, topologically closed, self-similar groups of tree automorphisms (thus, sets of tree automorphisms closed in each of the three settings).

Note that the topic of our choice sits in the intersection of three established areas of study: self-similar groups, closed (compact) groups of rooted tree automorphisms, and symbolic dynamics on rooted trees.

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## An introduction to totally disconnected locally compact groups

PHILLIP WESOLEK  Binghamton University

The study of locally compact groups splits into two cases: the connected groups and the totally disconnected groups. There is a rich and deep theory for the connected groups, which was developed over the last century. On the other hand, the study of totally disconnected locally compact groups only seriously began in the last 30 years, and moreover, these groups today appear to admit an equally rich and deep theory. In this minicourse, we will begin by presenting the basic theory of totally disconnected locally compact groups. We will then discuss examples related to trees and dynamics.

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# Research talks

## Colloquium

### Totally disconnected locally compact groups and actions on trees

George Willis   University of Newcastle Australia

Locally compact (l.c.) groups are fundamental to many areas of mathematics and their group structure is important in many of the contexts in which they appear. Each l.c. group divides into a connected factor and a totally disconnected factor. Understanding connected l.c. groups reduces, via the solution of Hilberts 5th Problem achieved in the 1950s, to the study of real Lie groups, but a comparable understanding of t.d.l.c. groups is only now being developed.

Part of this new understanding, analogous the role of eigenvalues and eigenvectors in Lie theory, is the concept of the scale on the t.d.l.c. group and associated actions of sub-quotients of the group on regular trees.

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## Invited talks

### Branch Groups and New Types of Subgroup Growth for Pro-$p$ Groups Branch Groups and Uncountable New Types of Subgroup Growth for Pro-$p$ Groups

Yiftach Barnea[1] and Jan-Christoph Schlage-Puchta[2]   [1]Royal Holloway, University of London, [2]Universität Rostock

In these couple of talks we will present joint work in which we used branch groups to construct new types of subgroup growth for pro-$p$ group.

In the first talk we will survey some general results on subgroup growth in pro-$p$ groups. A long standing open problem in this area is what is the subgroup growth of the Grigorchuk group and the Gupta-Sidki groups? We will show that a class of pro-$p$ branch groups including these examples have subgroup type $n^{\log n}$. We will then use action of direct product of these examples on direct product of trees to construct pro-$p$ groups with new type of subgroup growth, namely, $n^{(\log n)^k}$ for any positive integer $k$.

In the second talk, we will use some of the ideas from the first talk and actions of these examples on subtrees to construct pro-$p$ groups with subgroup growth of type $n^{(\log n)^{f(n)}}$ for slowly growing monotone functions $f(n)$. Thus, producing uncountable many new types of subgroup growth for pro-$p$ groups. This answers a well-known open problem by Lubotzky and Segal. Finally, we will present some open problems about such actions for which solutions could be useful in calculating more types.

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## Unitary representations of branch groups and non free actions

Rostislav Grigorchuk    Texas A&M University

I will describe a number of results obtained in three joint papers with Artem Dudko on unitary representations of weakly branch groups and absolute non-freeness of actions. This includes such types of representations as Koopman, quasiregular, and groupoid representations. It will be explained that groups of branch type have many pairwise not unitary equivalent representations of these types. On the other hand, in certain situations (in particular, in the case of actions on trees), they are weakly equivalent. The latter fact gives information about the spectral properties of representations and spectra of Cayley and Schreier graphs.

I will also discuss factor representations and a way to obtain them using absolutely non-free actions.

Indecomposable characters and invariant random subgroups will be mentioned in relation to these topics.

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## Uniformly recurrent subgroups and lattice embeddings

Adrien Le Boudec    CNRS/ENS Lyon

A uniformly recurrent subgroup of a group *Gamma* is a minimal subsystem of the space of subgroups Sub($\Gamma$). In this talk we will consider lattice embeddings within the class of countable groups defined by the property that the largest amenable URS is continuous. When this URS comes from an extremely proximal action, we obtain restrictions on the possible locally compact envelopes, notably regarding normal subgroups and product decompositions.

Time permitting, we might also discuss a specific family of groups within this class, which contain instances of finitely generated simple groups which act properly and cocompactly on the wreath product of a tree with a finite graph.

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## Morphisms between full groups

Nicolás Matté Bon    ETH Zürich

To any group or pseudogroup of homeomorphisms of the Cantor set one can associate a larger (countable) group, called the topological full group. It is known that every isomorphism between full groups of pseudogroups extends to a continuous isomorphism between the ambient pseudogroups. I will explain a necessary and sufficient condition under which an arbitrary homomorphism from a topological full group to another group of homeomorphisms extends to a continuous morphism of pseudogroups. If time permits, I will also explain an application related to a natural combinatorial fixed point property for group actions in terms of the growth of its orbits.

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## Highly-arc-transitive digraphs and connections with group theory

Rögnvaldur Möller    Háskóli Íslands

The class of highly-arc-transitive digraphs was defined by Cameron, Praeger and Wormald in an influential paper that appeared in 1993. In this talk, I will describe how graphs from this class regularly pop up in group theoretical work. I will also outline graph theoretical work to describe the structure of these graphs and resolve the many questions and conjectures made by Cameron, Praeger and Wormald.

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## Self-similarity of groups

SAID SIDKI    Universidade de Brasília

We will review self-similarity and virtual endomorphisms of groups. Then we follow with recent results on certain groups which do not admit faithful self-similarity, on new lamplighters and self-similar products of groups. These are based on joint works with A. Dantas, D. Kochloukova and L. Bartholdi.

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## Groups acting on biregular trees with prescribed local action

SIMON SMITH    University of Lincoln

I will begin this talk by describing a construction that generalises M. Burger and S. Mozes's iconic universal group $U(F)$ from regular trees to biregular trees. The construction is a group $U(M,N)$ that acts on a biregular tree $T$ which need not have finite valency. The action of $U(M,N)$ on $T$ is locally-$(M,N)$; that is, the stabiliser of any vertex $v$ induces either $M$ or $N$ on the neighbours of $v$. When M and N are transitive, the group $U(M,N)$ contains a permutationally isomorphic copy of every other locally-$(M,N)$ group, and so it is the universal locally-$(M,N)$ group. The groups $U(F)$ then arise as a special case of this construction, with $U(F)$ and $U(F, \mathrm{Sym}(2))$ isomorphic as topological groups.

I will then give an application of $U(M,N)$ to the study of permutation representations of locally compact groups that are totally disconnected ("tdlc groups").

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## An Ihara-type theorem for buildings

THOMAS WEIGEL    Università di Milano-Bicocca

As mentioned by J-P.Serre in the *Introduction* of his book *Trees*, Ihara's result identifying the affine Bruhat-Tits building of $\mathrm{SL}_2(K)$, $K$ a local field, with a regular tree, was the key result motivating him to start with a systematic study of group actions on trees. In the talk we want to introduce and study a class of buildings in which this original result of Ihara holds (in a somehow weaker form). By Bass-Serre theory this result leads to decomposition theorems for certain Kac-Moody groups and their subgroups.

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## Branch groups and their trees, and ordered sets

JOHN WILSON    University of Oxford/Universität Leipzig

From work of the speaker and Grigorchuk it is known that in many cases the spherically homogeneous tree $T$ on which a branch group acts can be recovered from the structure of $G$ as an abstract group. A very general method is proposed that makes the notion of 'recoverability' precise: $T$ has an explicit interpretation in $G$, in the sense of model theory. The context is the first-order theory of groups, and there are also applications to finite groups and to the study of the group $\mathrm{Aut}_O(\Lambda)$ of order automorphisms of a totally ordered set $\Lambda$: in particular, if $\mathrm{Aut}_O(\Lambda)$ acts transitively on $\Lambda$ and satisfies the same sentences as $\mathrm{Aut}_O(\mathbb{R})$ in the first-order language of group theory then $\Lambda$ is isomorphic to $\mathbb{R}$ as an ordered set.

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# Contributed talks

## Geodesic growth

Alex Bishop   University of Technology Sydney

The geodesic growth function of a group counts the number of geodesic words of a given length with respect to some finite generating set. In this talk, I will discuss some of the problems in the study of geodesic growth. In particular, I will present the open problem of the existence of a group with intermediate geodesic growth.

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## Groups of Piecewise Isometric Rearrangements of Tessellations

Robert Bieri   Univeristät Frankfurt

This is recent joint work with Heike Sach based on her Diploma Thesis (Frankfurt 1992)

We consider a tessellated piece $S$ of either Euclidean or hyperbolic n-space $X$ and are interested in rearranging the tiles of this tessellation by cutting $S$ along tile-boundaries in finitely many essentially convex rigid pieces and rearranging them to a new tessellation of $S$. Each such rearrangement defines a permutation of the tile-centers, and we call this a *piecewise Euclidean- isometric (pei)-permutation* (resp. a *piecewise hyperbolic-isometric (phi)-permutation*). We are interested in the corresponding permutation groups: $\mathrm{pei}(S)$ and $\mathrm{phi}(S)$, and also in subgroups like $\mathrm{pet}(S) \leqslant \mathrm{pei}(S)$, where the isometries are restricted to translations. All phi-, pei-, and pet-groups contain the normal subgroup $S_\infty$ consisting of all finite permutations.

Results:

1. If $X$ is the hyperbolic plane $\mathbb{H}^2$ (with the familiar $SL_2(\mathbb{Z})$-tessellation by triangles with one corner at infinity) then the quotient group $\mathrm{pei}(\mathbb{H}^2)/S_\infty$ is Thompson's group $V$.

2. When $X$ is Euclidean $\mathbb{E}^n$ and $S \subseteq \mathbb{E}^n$ is tessellated by unit cubes, then $\mathrm{pei}(S)$ and $\mathrm{pet}(S)$ have (not easy but) accessible finiteness properties and few normal subgroups. In particular, $\mathrm{pei}(\mathbb{E}^n)$ is of type $F_{2^n-1}$.

The conjunction of the two facts suggests that there is more waiting to be detected.

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## Automaton semigroups arising as free products

Tara Brough   Universidade Nova de Lisboa

Which free products of automaton semigroups are themselves automaton semigroups?

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## Inverse limits of finite-state automata and word problem for totally-disconnected locally-compact groups

Michal Ferov  University of Technology Sydney

In a finitely generated group, the formal language consisting of words representing the identity is called the word problem. In the setting of finitely generated groups, word problem is a classical and well-studied phenomenon. Our aim is to develop an analogue of the classical computer-scientific machinery in the setting of totally disconnected locally compact groups. The first step in this direction would be profinite automata - inverse limits of finite state automata and to showing that the in a finitely generated profinite group the set of sequences of words representing sequences of elements converging to the identity can be recognised by such object.

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## Grigorchuk-Gupta-Sidki groups as a source for Beauville surfaces

Şükran Gül  University of the Basque Country

Groups of automorphisms of regular rooted trees have received considerable attention over the past few decades after the first Grigorchuk group was designed to be a counterexample to the General Burnside Problem. Later on many different examples and generalizations came into the literature; for example the Gupta-Sidki groups and the second Grigorchuk group. The Grigorchuk-Gupta-Sidki groups (GGS-groups for short) are a family of groups generalizing them.

For every non-zero vector $\mathbf{e} = (e_1, \dots, e_{p-1}) \in \mathbb{F}^{p-1}$ there corresponds a GGS-group $G = \langle a, b \rangle$, where $a$ is a rooted automorphism defined by the permutation $(12 \dots p)$ and $b$ is recursively defined according to the given vector $\mathbf{e}$. Vovkivsky showed that $G$ is always infinite and it is a periodic group if and only if $\sum_{i=1}^{p-1} e_i = 0$, so some of them are also counterexamples for the General Burnside Problem. For these groups there is a very natural family of normal subgroups of finite index, which are the level stabilizers $\mathrm{st}_G(n)$ for each $n \in \mathbb{N}$. The aim of this talk is to study the existence of Beauville surfaces associated to these quotients.

Roughly speaking, a *Beauville surface* is a compact complex surface defined by taking a pair of complex curves $C_1$ and $C_2$ and letting a finite group $G$ act freely on their product to define the surface as the quotient $(C_1 \times C_2)/G$. A finite group $G$ giving rise to such a surface is called a *Beauville group*. These groups can be described in purely group theoretical terms. A finite group $G$ is called a Beauville group if it is a 2-generator group and there exists a pair of generating sets $\{x_1, y_1\}$ and $\{x_2, y_2\}$ of $G$ such that $\sum(x_1, y_1) \cap \sum(x_2, y_2) = 1$, where for $i = 1, 2$

$$\sum(x_i, y_i) = \bigcup_{g \in G} \left( \langle x_i \rangle^g \cup \langle y_i \rangle^g \cup \langle x_i y_i \rangle^g \right).$$

Note that if $G$ is a GGS-group, then the quotients $G/\mathrm{st}_G(n)$ are finite $p$-groups generated by two elements of order $p$. Thus, they are natural candidates to search for Beauville $p$-groups. It turns out that the property of being Beauville for these quotients depends on whether $G$ is periodic or not. In this talk, we prove the following: if $G$ is periodic then the quotients $G/\mathrm{st}_G(n)$ are Beauville groups for every $n \geq 2$ if $p \geq 5$ and $n \geq 3$ if $p = 3$. On the other hand, if $G$ is non-periodic, then none of the quotients $G/\mathrm{st}_G(n)$ are Beauville groups. Thus, a periodic GGS-group is a source for the construction of an infinite series of Beauville $p$-groups. This gives yet another reason why GGS-groups constitute an important family in group theory.

This is joint work with Jone Uria-Albizuri

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## Almost automorphisms of trees and completions of Thompson's $V$

Waltraud Lederle    Ben Gurion University of the Negev

Caprace and De Medts discovered that Thompson's $V$ can be written as a group of tree almost automophism group, allowing to embed it densely into a totally disconnected, locally compact (t.d.l.c.) group. Matui discovered that it can be written as the topological full group of the groupoid associated to a one-sided shift. Combining these, we find countably many different t.d.l.c. groups containing a dense copy of $V$.

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## Weakly maximal subgroups and actions on tree

Paul-Henry Leemann    ENS Lyon

Weakly maximal subgroups (subgroups maximal among those of infinite index) are a natural generalization of maximal subgroups. Bartholdi and Grigorchuk showed that for a branch action, all stabilizers of rays are weakly maximal, infinite and distincts subgroups. Later, with Bou-Rabee and Nagnibeda we proved that in general branch groups have a lot of weakly maximal subgroups not coming from any branch action. I will explain how such subgroups may be used to construct new (not branch) action on rooted trees and some properties of these actions.

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## When are GGS-groups conjugate?

Moritz Petschick    Heinrich-Heine Universität Düsseldorf

GGS-groups on the $p$-adic tree are defined by some tuple of length $p-1$ consisting of integers modulo $p$, called the defining tuple. Two GGS-groups are conjugate in the automorphism group of the respective tree if and only if their defining tuples can be transformed into each other by multiplication by some integer modulo $p$ and certain reorderings of the tuple. Knowing this, the number of non-conjugate GGS-groups can be calculated.

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## Some properties of group actions on zero-dimensional spaces

Colin Reid    University of Newcastle Australia

Let $G$ be a group acting on a topological space $X$. The orbit closure relation $R$ on $X$ is defined by setting $(x, y) \in R$ if $y$ lies in the closure of the $G$-orbit of $x$. In the case that $G$ is finitely generated and $X$ is compact metrizable zero-dimensional, it was shown by AuslanderGlasnerWeiss that $R$ is symmetric if and only if it is a closed subset of $X \times X$. In particular, it follows that the action is distal if and only if it is equicontinuous. I will discuss a generalization of this result to actions of compactly generated groups on locally compact zero-dimensional spaces, and illustrate how it can be used in the structure theory of totally disconnected locally compact groups via actions on coset spaces.

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## On Growth of Generalized Grigorchuk's Overgroups

Supun Samarakoon    Texas A&M University

Grigorchuk's Overgroup $\tilde{G}$, is a branch group of intermediate growth. It contains the first Grigorchuk's torsion group $G$ of intermediate growth constructed in 1980, but also has elements of infinite order. It's growth is substantially greater than the growth of $G$. The group $G$, corresponding to the sequence $012012\ldots$, is a member of the family $\{G_\omega\}, \omega \in \Omega = \{0,1,2\}^{\mathbb{N}}$ consisting of groups of intermediate growth when sequence $\omega$ is not virtually constant. Following this construction we define generalized overgroups $\{\tilde{G}_\omega, \omega \in \Omega\}$ such that $G_\omega$ is a subgroup of $\tilde{G}_\omega$ for each $\omega \in \Omega$. We prove, if $\omega$ is eventually constant, then $\tilde{G}_\omega$ is of polynomial growth and if $\omega$ is not eventually constant, then $\tilde{G}_\omega$ is of intermediate growth. As a subset of the space $\mathcal{M}_8$ of marked groups with eight generators, the set $\{\tilde{G}_\omega, \omega \in \Omega\}$ of generalized overgroups is not complete. We describe the completion of it and explain a similarity and a difference with the completion of the classical Grigorchuk's family $\{\tilde{G}_\omega, \omega \in \Omega\}$.

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## Galois coverings of Schreier graphs of groups generated by bounded automata

Asif Shaikh R. A. Podar College Mumbai 19 / Pune University 7

Joint work with Daniele DAngeli, Hemant Bhate and Dilip Sheth.

I shall present a characterization of the covering Schreier graphs of groups generated by bounded automata to be Galois. We investigate the zeta and $L$ functions of Schreier graphs of these groups. We also investigate the zeta and $L$ functions of zig-zag products of Schreier graphs of Basilica group and 4 cycle graph.

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## Generating lamplighter-like groups with bireversible automata

Rachel Skipper Binghapmton University

We use the language of formal power series to construct finite state automata generating groups of the form $A \wr \mathbb{Z}$ where $A$ is the additive group of a finite commutative ring. We then provide conditions on the ring and the power series which make automata bireversible.

This is a joint project with Ben Steinberg.

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## On generalised multi-edge spinal groups.

Anitha Thillaisundaram University of Lincoln

We will investigate a generalisation of the Grigorchuk-Gupta-Sidki branch groups and talk about their maximal subgroups and about their profinite completion. Additionally, we demonstrate a link to a conjecture of Passman on group rings.

This is joint work with Benjamin Klopsch.

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## Groups acting on trees with non-trivial quasi-center

Stephan Tornier University of Newcastle Australia

Groups acting on trees form a particularly accessible and important class of totally disconnected, locally compact groups. Burger–Mozes have developed a structure theory of closed, non-discrete and locally quasiprimitive subgroups of the automorphism group of a regular tree which resembles the theory of semisimple Lie groups. The collection of elements with open centralizer in such a group, termed the quasi-center, features prominently in this theory. We present a theorem which characterizes the types of automorphisms which the quasi-center of a non-discrete subgroup of the automorphism group of a regular tree may contain in terms of its local action. Explicit construction of such groups with non-trivial quasi-center shows that our assertions are sharp.

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## Invariant random subgroups in groups acting on rooted trees.

László Márton Tóth Alfréd Rényi Institute of Mathematics Budapest

An invariant random subgroup (IRS) of a countable group is a conjugation invariant probability distribution on the space of subgroups. There has been a number of recent papers studying IRS's in various types of groups: lattices in Lie groups, the group of finitary permutations of a countable set, free groups, lamplighters and more.

In this talk I will consider IRS's in branch groups, in particular the group of finitary automorphisms of a d-ary rooted tree. We exploit the action of these groups on the boundary of the tree to understand fixed point sets of ergodic IRS's. We show that in the fixed point free case IRS's behave like the ones in lattices in Lie groups, but if there are fixed points they resemble the ones in the finitary permutation group.

Joint work with Ferenc Bencs.

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## On self-similar finite $p$-groups.

Matteo Vannacci   Heinrich-Heine Universität Düsseldorf

Self-similar groups are a special class of groups acting faithfully on a rooted tree. These naturally arise as counterexamples to a number of problems in Group Theory, for example the Grigorchuk groups and Gupta-Sidki groups are self-similar. In this talk, after introducing the main definitions, I will describe how this class is "very rigid". For instance, one can show that the number of self-similar finite $p$-groups of rank $r$ is finite and bounded by a function of $p$ and $r$. Moreover, it is possible to fully characterise self-similar $p$-groups of maximal class.

This is joint work with A. Babai, K. Fathalikhani and G. A. Fernandez-Alcober.

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## Automorphism group of universal Coxeter group

Olga Varghese   WWU Münster

We study geometric properties of the automorphism group of universal Coxeter group of rank $n \geq 4$, $\mathrm{Aut}(W_n)$. We prove that $\mathrm{Aut}(W_n)$ has Serre's fixed point property FA., but it does not have Kazhdans property (T).

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