Notizen:

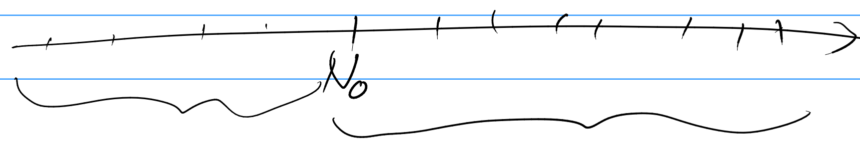
$$a_n = (-1)^n \cdot \frac{1}{n} \xrightarrow{n \to \infty} 0$$

$$\sum_{n=1}^{\infty} (-1)^n a_n = \sum (-1)^n \cdot (-1)^n \cdot \frac{1}{n} = \underbrace{\sum \frac{1}{n}}_{\text{harmonische Reihe}} \text{ divergiert}$$

$\underline{\sum \frac{1}{n} \text{ divergent}}$, $a_n = \frac{1}{n} \rightsquigarrow |a_{n+1}| \leq \overset{q=1}{1} \cdot |a_n|$

$$\frac{1}{n+1} \leq \frac{1}{n} \quad ✓$$

(a_n) kgt. gegen $c \in \mathbb{R} \iff \forall \varepsilon > 0$ $\boxed{\exists N_0 \in \mathbb{N}\ \forall n \geq N_0 : |a_n - c| \leq \varepsilon}$



$\forall \varepsilon > 0$: Es gibt nur endl. viele $n \in \mathbb{N}$ mit $|a_n - c| > \varepsilon$

$A(n)$ gilt für endl. viele $n \in \mathbb{N}$

$\iff \exists N_0 \in \mathbb{N}\ \forall n \geq N_0 : \neg A(n)$

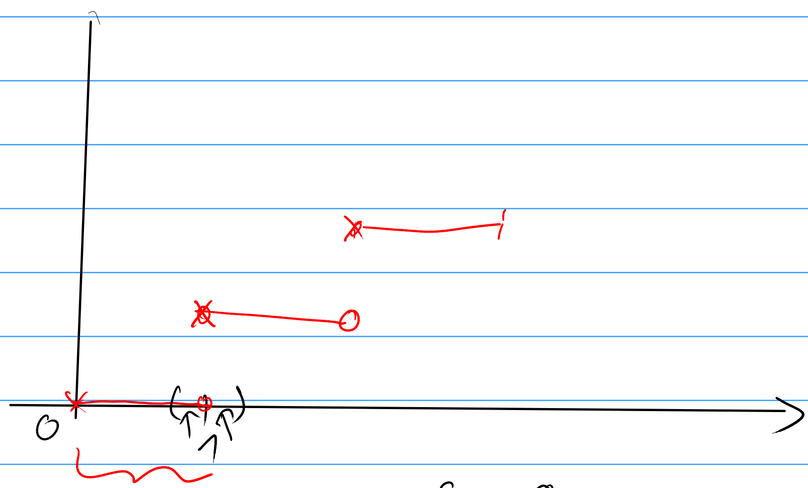
$$\boxed{\sum_{n=0}^{\infty} \frac{n!}{(2n)!}}\,, \qquad a_n = \frac{n!}{(2n)!}$$

Quot. Krit.: $$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(2n+2)!} \cdot \frac{(2n)!}{n!} = \frac{n+1}{(2n+1)(2n+2)}$$

$$= \frac{1+\frac{1}{n}}{(2+\frac{1}{n})(2n+2)} = \underbrace{\frac{\frac{1}{n}+\frac{1}{n^2}}{(2+\frac{1}{n})(2+\frac{2}{n})}}_{\to 0}$$

Für $0<q<1$ und alle hinr. großen n

gilt somit: $\frac{a_{n+1}}{a_n} \leq q < 1$

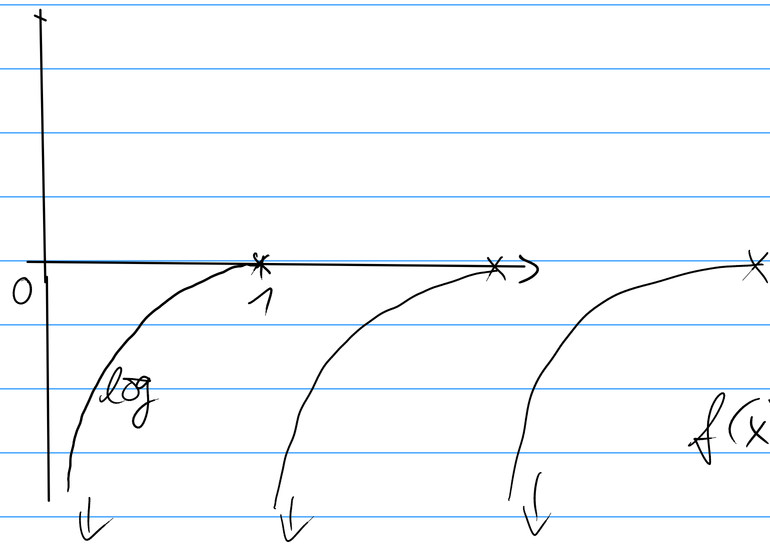


$\frac{\varepsilon}{2} = 0{,}5$

$|x-0| < \frac{\varepsilon}{2}$

$$\left.\begin{array}{l} f: \mathbb{Z} \to \mathbb{R} \\ \quad n \mapsto n \end{array}\right\} \text{stetig } \checkmark$$

Unbeschr. + 1-periodische Fkt.:



$$f(x) = \log(x - \lfloor x \rfloor)$$

$f: \mathbb{R} \to \mathbb{R}$ heißt stetig

$$\Leftrightarrow \forall x \in \mathbb{R}: \boxed{\forall} (x_n): \left(x_n \xrightarrow[n\to\infty]{} x \Rightarrow \underline{\underline{f(x_n) \xrightarrow[n\to\infty]{} f(x)}} \right)$$

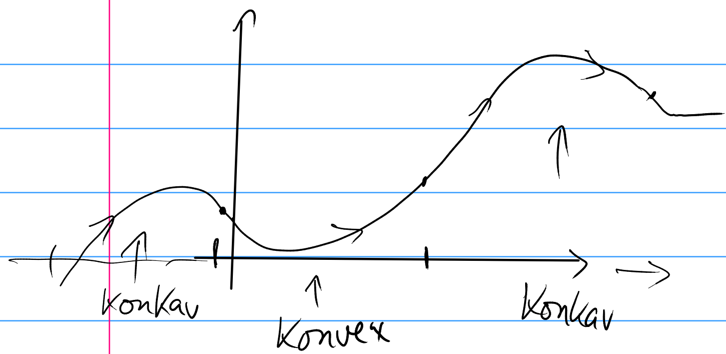
$(x_n) = \left(x + \frac{1}{n}\right)_{n \in \mathbb{N}}$

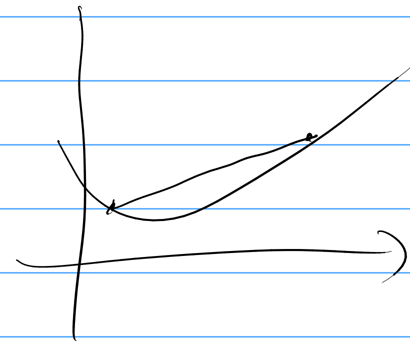
$(x_n) = \left(x - \frac{1}{n}\right)_{n \in \mathbb{N}}$

Bsp. für unstetige Fkt. f mit Krit. ⊛,
d.h. mit $\lim f(x - \frac{1}{n}) = \lim f(x + \frac{1}{n}) = f(x)$:

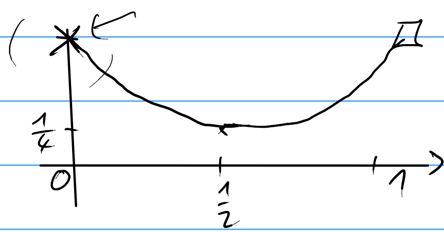
$$f(x) = \begin{cases} 0, & x = 0 \\ \frac{1}{n}, & x = \frac{1}{n}, n \in \mathbb{N} \\ -\frac{1}{n}, & x = -\frac{1}{n}, n \in \mathbb{N} \\ 1, & \text{sonst} \end{cases}$$

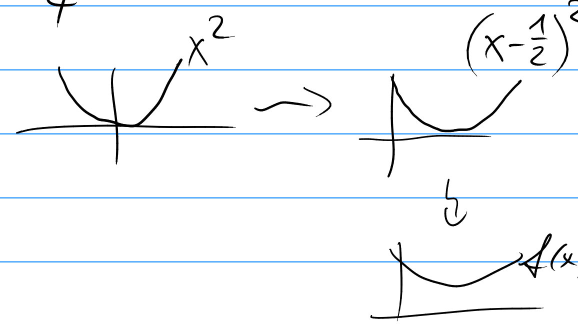
ist unstetig:
Betr. $x_n = \frac{2}{2n+2} \to 0$





$$f(x) = x^2 - x + \frac{1}{2} = \left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$$





Regel von de l'Hospital:

Sei $f, g :]a, b[\to \mathbb{R}$ diff'bar, $g'(x) \neq 0$ für alle $x \in]a,b[$,

und sei $\lim\limits_{x \to a+} f(x) = \lim\limits_{x \to a+} g(x) = 0$.

Dann:
$$\lim_{x \to a+} \frac{f(x)}{g(x)} = \lim_{x \to a+} \frac{f'(x)}{g'(x)}.$$

Bsp.: $$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = 1$$

$$\frac{f(x)}{g(x)} = \frac{\frac{1}{g(x)}}{\frac{1}{f(x)}}$$

$$\left(\frac{\sin x}{x}\right)^{3/x^2} = \exp\Big(\underbrace{\frac{3}{x^2}\log\left(\frac{\sin x}{x}\right)}_{=:f(x)}\Big) \longrightarrow e^{-1/2}$$

↑ Stetigkeit von exp

<u>Beh.:</u> $f(x) = \dfrac{3\log\left(\frac{\sin x}{x}\right)}{x^2} \xrightarrow{x\to 0} -\dfrac{1}{2}$

<u>Bew.:</u> de l'H:

$$\frac{3\cdot\frac{x}{\sin x}\cdot\frac{x\cos x-\sin x}{x^2}}{2x} = \frac{3}{2}\cdot\underbrace{\frac{x\cos x-\sin x}{x^2\sin x}}$$

de l'H:

$$\frac{3}{2}\cdot\frac{-\sin x}{2\sin x + x\cos x} = \frac{3}{2}\cdot\frac{-1}{2+\underbrace{\frac{x}{\sin x}}_{\to 1}\cdot\underbrace{\cos x}_{\to 1}}$$

$$\xrightarrow{x\to 0} \underline{\underline{-\frac{1}{2}}}$$

$\square$