Mathematisches Institut
Heinrich-Heine-Universität Düsseldorf
Prof. Dr. Stefan Schröer WS 21/22

# Algebraic Geometry I

## Sheet 4

**Excercise 1.** Let $L$ be an ordered set, viewed as category, and $L \to (\mathrm{Ab})$ be a contravariant functor, comprising abelian groups $G_\lambda$, $\lambda \in L$ and *transition maps* $f_{\lambda\mu} : G_\mu \to G_\lambda$, $\lambda \leq \mu$. On the disjoint union $\bigcup_{\lambda \in L} G_\lambda$, we consider the relation

$$a_\lambda \sim a_\mu \quad \Longleftrightarrow \quad f_{\lambda\eta}(a_\lambda) = f_{\mu\eta}(a_\mu) \text{ for some } \eta \geq \lambda, \mu.$$

Assume that the ordered set $G$ is *directed*, that it, for each $\lambda, \mu \in L$ there is some $\eta \in L$ with $\lambda, \mu \leq \eta$. Check that the above is an equivalence relation, and that the set of equivalence classes

$$\varinjlim_{\lambda \in L} G_\lambda = (\bigcup_{\lambda \in L} G_\lambda)/\sim$$

inherits the structure of an abelian group. Furthermore, interpret localizations $S^{-1}R$ and stalks $\mathscr{F}_a$ as such *direct limits*.

**Exercise 2.** Let $(X, \mathscr{O}_X)$ be a ringed space, and $\mathscr{F}$ be a presheaf of modules. Show that the sheafification $\mathscr{F}^+$, whose groups of local sections $\Gamma(U, \mathscr{F}^+)$ comprises the compatible tuples

$$(s_a)_{a \in U} \in \prod_{a \in U} \mathscr{H}_a,$$

indeed satisfies the sheaf axiom.

**Exercise 3.** Let $X = \mathbb{A}^1$ be the affine line over some ground field $k$. Give an example of a non-zero presheaf of modules $\mathscr{F}$ whose sheafification $\mathscr{F}^+$ becomes the zero sheaf.

**Exercise 4.** Let $\mathscr{F}$ and $\mathscr{G}$ be $\mathscr{O}_X$-modules on some ringed space $X$.

(i) Define the tensor product sheaf $\mathscr{F} \otimes_{\mathscr{O}_X} \mathscr{G}$.

(ii) Suppose $X$ is a scheme, with $\mathscr{F}$ and $\mathscr{G}$ quasicoherent. Show that the tensor product sheaf $\mathscr{F} \otimes_{\mathscr{O}_X} \mathscr{G}$ is quasicoherent as well.

**Abgabe:** Bis Donnerstag, den 18. November um 23:55 Uhr über ILIAS.