

T. Brzeziński and R. Wisbauer: Corings and Comodules (CUP, 2003). Erratum

- page 11, 2.8: The phrase “provided $\text{Im } f$ is a pure submodule of C ” should be added.
- page 18, exercise 2.15.3, penultimate line: Replace “an algebra morphism” by “a coalgebra morphism”;
- page 20, line 10: “the dual coalgebra” should be “the dual algebra”;
- page 23, lines -4/-3: to derive coassociativity of ϱ^K it does not suffice to require f to be C -pure - it should be $C \otimes_R C$ -pure (which implies C -pure, since C is a direct summand of the R -module $C \otimes_R C$); similar refinements are needed at other places.
- page 24, line 10: “if K is a C -pure R -submodule of M ” should read “if K is a $C \otimes_R C$ -pure R -submodule of M ”.
- page 24, lines 12/13: “ f is a C -pure morphism” should read “ f is a $C \otimes_R C$ -pure morphism”.
- page 28, section 3.12, proof of (3): second term in the equation has a misplaced parenthesis;
- page 33: In the proof of 3.18(3), $M \otimes_A C$ should be $M \otimes_R C$.
- page 43, line 2: “ ${}^C\mathbf{M}$ ” should be “ \mathbf{M}^C ”.
- page 43, line -3; page 44, line 1,2: “ ϱ_M ” should be “ ϱ^M ”.
- page 47, section 4.9, proof: In the first displayed formula, $f \otimes p$ should be replaced by fp (thrice).
- page 47, line -15: It should be “ ${}^{P^*}\varrho : P^* \rightarrow (P^* \otimes_R P) \otimes_R P^*$ ”.
- page 53, section 4.17: (b) should be: $I = \text{Ann}_{C^*}(W)$ for some $W \subset N \in \mathbf{M}^C$; (i) should read: ...in the (right) C -adic topology.
- page 61, Exercise 5.13: in (4)(i), delete “(free)”.
- page 63 in 6.4(d): replace “ $\beta^r * f(c)$ ” by “ $\beta^r(c) * f$ ”.
- page 79, Exercise 8.12: in (1)(ii): read “subcoalgebra”; in (1)(ii), (2)(i) and (iii): assume R to be a field.

- page 85, line 9: “subalgebras” should be “subcoalgebras”.
- page 102, line -11: “ $\omega_{M,N}$ is a D -pure morphism” should read “ $\omega_{M,N}$ is a $D \otimes_R D$ -pure morphism”.
- page 102, line -8: “ $\omega_{M,N}$ is D -pure” should read “ $\omega_{M,N}$ is $D \otimes_R D$ -pure”.
- page 102, line -7: “ $\omega_{L,M}$ is C -pure” should read “ $\omega_{L,M}$ is $C \otimes_R C$ -pure”.
- page 102, line -5: “ $\omega_{M,N}$ is D -pure and B -pure” should read “ $\omega_{M,N}$ is $D \otimes_R D$ -pure and $B \otimes_R B$ -pure”.
- page 182, line 18: “ f is \mathcal{C} -pure as a right A -morphism” should read “ f is $\mathcal{C} \otimes_A \mathcal{C}$ -pure as a right A -morphism”.
- page 182, lines 23 & 26: “ \mathcal{C} -pure” should read “ $\mathcal{C} \otimes_A \mathcal{C}$ -pure”.
- page 187, line -8: \mathbf{M}^C should be replaced by \mathbf{M}^C .
- page 189, line 14: “coalgebra” should be “coring”.
- page 191, line -4: “(b) \Rightarrow (c)” should be replaced by “(d) \Rightarrow (c)”.
- page 208: Statement (2) in 19.22 is incorrect and should be removed.
- page 213: In 20.5, the statement: *If \mathbf{M}^C is closed under essential extensions, then Rat^C is exact* is not true and should be removed.
- page 220: In 21.6 and 21.7, “ ${}_A\mathcal{C}$ flat” should be “ \mathcal{C}_A flat”.
- page 224, line 10: “ $\omega_{M,N}$ is a \mathcal{D} -pure morphism” should read “ $\omega_{M,N}$ is a $\mathcal{D} \otimes_B \mathcal{D}$ -pure morphism”.
- page 224, line 14: “ $\omega_{M,N}$ is \mathcal{D} -pure” should read “ $\omega_{M,N}$ is $\mathcal{D} \otimes_B \mathcal{D}$ -pure”.
- page 224, line 15: “ $\omega_{L,M}$ is \mathcal{C} -pure” should read “ $\omega_{L,M}$ is $\mathcal{C} \otimes_A \mathcal{C}$ -pure”.
- page 224, lines 17/18: “ $\omega_{M,N}$ is \mathcal{D} -pure in ${}_B\mathbf{M}$ and \mathcal{D}' -pure in $\mathbf{M}_{B'}$ ” should read “ $\omega_{M,N}$ is $\mathcal{D} \otimes_B \mathcal{D}$ -pure in ${}_B\mathbf{M}$ and $\mathcal{D}' \otimes_{B'} \mathcal{D}'$ -pure in $\mathbf{M}_{B'}$, and $I_D \otimes \omega_{M,N}$ is \mathcal{D}' -pure in $\mathbf{M}_{B'}$ ”.
- page 230, 23.1: In 23.1 the assumptions: for all right \mathcal{C} -comodules, the right B -module map $\varrho^M \otimes I_{F(C)} - I_M \otimes {}^{F(C)}\varrho$ is $\mathcal{D} \otimes_B \mathcal{D}$ -pure and F preserves kernels should be added. A sufficient condition for the former is that ${}_B\mathcal{D}$ is a flat module.

- page 231: In 23.2 statement (5) only holds under additional conditions (which imply associativity of the tensor products concerned).
- page 237: In 23.10 the conditions on Y include that it is faithfully coflat in $\mathbf{M}^{\mathcal{C}}$; this is not necessary to get the equivalence. It is sufficient (and necessary) to require that Y is a (B, \mathcal{C}) -bicomodule that is (B, \mathcal{C}) -quasifinite and a (B, \mathcal{C}) -injector-cogenerator (see corrections for 23.12).
- page 238, line 17: “faithfully flat” should be “faithfully coflat”.
- page 238: In 23.12, (a) does not imply the conditions (b),(c) (not covered by 23.11); they should be replaced by
 - (b) there exists a $(\mathcal{D}, \mathcal{C})$ -bicomodule Y that is (B, \mathcal{C}) -quasifinite, a (B, \mathcal{C}) -injector-cogenerator, and $e_{\mathcal{C}}(Y) \simeq \mathcal{D}$ as corings;
 - (c) there exists a $(\mathcal{C}, \mathcal{D})$ -bicomodule X that is (A, \mathcal{D}) -quasifinite, an (A, \mathcal{D}) -injector-cogenerator, and $e_{\mathcal{C}}(Y) \simeq \mathcal{D}$ as corings.

where Y is a (B, \mathcal{C}) -injector-cogenerator means that for any injective cogenerator Q in \mathbf{M}_B , $Q \otimes_B Y$ is an injective cogenerator in $\mathbf{M}^{\mathcal{C}}$.
- page 239: Proof (a) \Leftrightarrow (b): It follows from the defining isomorphism that Y is a (B, \mathcal{C}) -injector-cogenerator if and only if the functor $h_{\mathcal{C}}(Y, -)$ is faithful and exact, i.e., $h_{\mathcal{C}}(Y, \mathcal{C})$ is faithfully coflat as left \mathcal{C} -comodule. This implies that Y is faithfully coflat as left \mathcal{D} -comodule and then essentially the proof of 23.10 can be followed.
- page 243, item 24.8: In the definition of a pure morphism of corings the map $\omega_{N, B \otimes_A \mathcal{C}}$ should be required to be $\mathcal{C} \otimes_A \mathcal{C}$ -pure. The subsequent sentence needs obvious adaption.
- page 246, line -2: $\text{Hom}^{\mathcal{C}}(M, B \otimes_A \mathcal{C})$ should read $\text{Hom}^{\mathcal{C}}(M, B \otimes_A \mathcal{C})$.
- page 320, 31.25: It can be shown that any left \mathcal{H} -comodule M can be equipped with a unique right A -module structure such that $\text{Im}(M_{\varrho}) \subseteq \mathcal{H} \times_A M$ (cf. page 240 in [G. Böhm, *Galois theory for Hopf algebroids*, Ann. Univ. Ferrara - Sez. VII - Sc. Mat. 51: 233–262 (2005)]), hence any left \mathcal{H} -comodule algebra is strict.
- page 335, 33.1: In 33.1, B should be required to be a faithful R -module; this is needed to prove that if ψ is an entwining, then B is a bialgebra, in particular to show that $\varepsilon(1_B) = 1_R$.
- page 366, line 8: “an R -module B ” should read “a *faithful* R -module B ” (i.e., in the whole of Section 36 it is assumed that B is a faithful R -module).

- page 382, 37.1 and pages 385–386, 37.8: The definition of a weak entwining structure is equivalent to the definition of a self-dual weak entwining structure. I.e. condition (we.2) in 37.1 should be replaced by

$$\sum_{\alpha} a_{\alpha} \otimes c^{\alpha}_{\underline{1}} \otimes c^{\alpha}_{\underline{2}} = \sum_{\alpha, \beta} a_{\beta\alpha} \otimes c_{\underline{1}}^{\alpha} \otimes c_{\underline{2}}^{\beta}.$$

- page 386 line 2: The formula (S) should read:

$$\sum_{\alpha} a_{\alpha} \otimes c^{\alpha}_{\underline{1}} \otimes c^{\alpha}_{\underline{2}} = \sum_{\alpha, \beta} a_{\beta\alpha} \otimes c_{\underline{1}}^{\alpha} \otimes c_{\underline{2}}^{\beta}. \quad (S)$$

- page 392 line 16: “ $b_{\underline{0}S_B}(\underline{\varepsilon}_C(c))b'$ ” should be replaced by “ $bs_B(\underline{\varepsilon}_C(c))b'$ ”.
- page 399, section 38.15, start of line 4: “ $f\pi_{\lambda} = f_{\lambda}$ ” should be replaced by “ $\pi_{\lambda}f = f_{\lambda}$ ”;
- page 409, section 39.1, claim (2): The assumption “and η_T is an isomorphism” should be added;
- page 409, section 39.1, proof, line after second display: “... is exact in **Ab**” should read “is exact in **A**”;
- page 413, section 39.6, proof, line 2: “the action on $F(A)$ is given by $F(\phi_A)$ ” should be “the action on $F(A)$ is given by $F_{A,A} \circ \phi_A$ ”;
- page 414, last displayed formula: The left hand $\Psi_{C,C,A}$ should be $\Psi_{C \otimes_T C, A}$;
- page 420, line 14: “a sequence” should be “an exact sequence”.
- page 423, line 10: “ $g(m)(1)$ ” should be “ $g(-)(1)$ ”.
- page 423, section 40.22, first line of the proof: “counit” should be “unit”.
- page 435, section 42.3, page 436, line 1: in (b) should be: $I = \text{Ann}_A(W)$ for some $W \subset N \in \sigma[M]$.
- page 436, section 42.4: In (b) and (c), N should be replaced by M .
- page 436, line -14: It should be “Choose $t_k \in T$ such that $t_k m_k = m_k$ and $a_i = m_i - t_k m_i$ ”.
- page 437, proof of 42.5: In the bottom row of the diagram, ${}_A\text{Hom}(L, M)$ should read ${}_A\text{Hom}(L, N)$.
- page 438, line 4: “ N -dense” should read “ M -dense”.
- page 442, line 9: replace “left exactness” by “exactness”.

- page 443, line 1-6: replace some " T " by " \tilde{T} ".
- page 444: Statement (d) in 42.19 is not equivalent to the statements (a)–(c), only the implication (c) \Rightarrow (d) holds. The (c) \Leftrightarrow (d) part of the proof should be removed.
- page 445: in 43.2(2)(a) replace "left ideal" by "right ideal".
- page 446: in 43.5(e) assume M to be self-projective.
- page 452: in 44.5(e) replace " $\mathcal{T}^{N\lambda}$ " by " $\mathcal{T}^{M\lambda}$ ".
- page 453: In 44.6, (4) can be deleted, it is equal to (3).
- page 454: line -11: assume $\lambda \neq \mu$.

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**Reader, please inform about any mistakes either:
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Thank you.